University of Freiburg
Dept. of Computer Science
Prof. Dr. F. Kuhn
P. Schneider

# Advanced Algorithms
# Problem Set 5

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## Exercise 1: Multiplicative Weight Update with Switches

Given $n$ experts, in each round one must choose (the advice of) one expert. There are $T$ rounds in total and $T$ is know in advance. Choosing expert $i$ in round $t$ causes a loss of $f_i^t$ and $|f_i^t| \leq 1$. We saw how to use multiplicative weight updates (MWU) to select experts, such that the *regret*, defined as difference between the expected loss of MWU and the optimal strategy, is in $O\left(\sqrt{T \ln n}\right)$.
Assume that we want to compare MWU to the best strategy which is allowed to switch from one expert to another at most $k$ times (instead of comparing MWU to just the best expert). In the rounds between switches the latest chosen expert is reused. The minimal loss with $k$ switches is given by

$$\min_{1=t_1<\ldots<t_k<t_{k+1}=T+1} \left( \sum_{j=1}^{k} \min_{1\leq i\leq n} \sum_{t=t_j}^{t_{j+1}-1} f_i^t \right).$$

Adapt the MWU strategy (where we are allowed to switch each round), such that it has a regret of at most $O\left(\sqrt{Tk \ln(nT)}\right)$ compared to the optimal strategy with at most $k$ expert switches. Then prove your claim. For which range of $k$ does the *average* regret converge to zero, as $T \to \infty$?