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Advanced Algorithms
Problem Set 9

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Exercise 1: Counting Cuts

It is known that a graph with edge connectivity at least λ contains at least $\lambda/2$ edge disjoint spanning trees (a result from Tutte and Nash-Williams from the 1960s). Use this result to show that there are at most $O(\lambda n^{2\alpha})$ cuts of size at most $\alpha\lambda$.

Exercise 2: Approximating Cuts in Graphs with Large Expansion

Let $G = (V, E)$ be an unweighted graph for which the following property holds. For all cuts S (w.l.o.g. we assume $|S| \leq |E \setminus S|$ otherwise we switch the roles of S and $E \setminus S$) and some (large) constant α we have that $e(S)/|S| \geq \alpha$. Show that by sampling edges with probability $p := \min(\frac{c \ln n}{\alpha\varepsilon^2}, 1)$ and assigning appropriate weights, w.h.p. we obtain a subgraph with $\tilde{O}(|E|/\alpha)$ edges that is an $(1 \pm \varepsilon)$-approximation of all cuts (for constant $\varepsilon > 0$).