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Algorithms and Data Structures
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Exercise Sheet 4

Exercise 1: Universal Hashing[1]

Consider a hashtable of size $m = 11$ and let $p = 101$. Consider hash functions of the form $h_{a,b}(x) := \big[(ax+b) \bmod p\big] \bmod m$, which form a $\approx$ 1-universal family[2] $\mathbb{H}_{a,b} = \{h_{a,b} \mid a, b \in \{1, ..., p–1\}\}$. Choose one hash function h from the family $\mathbb{H}_{a,b}$. Then find five *different* keys from the set $\mathbb{U} = \{0, \ldots, 99\}$, such that all keys are mapped to the same table entry. Then select a hash function h' from the family $\mathbb{H}_{a,b}$ randomly (or invent numbers a, b) and remap all keys into the table.

Exercise 2: Hashing with Open Addressing - Examples

(a) Let $h(s, j) := h_1(s) - 2j \bmod m$ and let $h_1(x) = x + 2 \bmod m$. Insert the keys 51, 13, 21, 30, 23, 72 into the hash table of size $m = 7$ using linear probing for collision resolution (the table should show the final state).

0	1	2	3	4	5	6

(b) Let $h(s, j) := h_1(s) + j \cdot h_2(s) \bmod m$ and let $h_1(x) = x \bmod m$ and $h_2(x) = 1 + \big(x \bmod (m-1)\big)$. Insert the keys 28, 59, 47, 13, 39, 69, 12 into the hash table of size $m = 11$ using the double hashing probing technique for collision resolution. The hash table below should show the final state.

0	1	2	3	4	5	6	7	8	9	10

(c) Repeat part (a) using the *"ordered hashing"* optimization from the lecture.

(d) Repeat part (b) using the *"Robin-Hood hashing"* optimization from the lecture.

[1] The exercise was changed a bit in order to get a more distinctive result
[2] For $p \gg m$ both prime.