



## Algorithms and Data Structures Summer Term 2019 Exercise Sheet 6

### Exercise 1: Master Theorem for Recurrences

Use the *Master Theorem* for recurrences, to fill the following table. That is, in each cell write  $\Theta(g(n))$ , such that  $T(n) \in \Theta(g(n))$  for the given parameters  $a, b, f(n)$ . Assume  $T(1) \in \Theta(1)$ . Additionally, in each cell note the case you used (1st, 2nd or 3rd by the order given in the lecture). We filled out one cell as an example.

$T(n) = aT(\frac{n}{b}) + f(n)$	$a = 16, b = 2$	$a = 1, b = 2$	$a = b = 3$
$f(n) = 1$	$\Theta(n^4)$ , 1st		
$f(n) = n$			
$f(n) = n^4$			

### Exercise 2: Peak Element

You are given an array  $A[1 \dots n]$  of  $n$  integers and the goal is to find a peak element, which is defined as an element in  $A$  that is equal to or bigger than its direct neighbors in the array. Formally,  $A[i]$  is a peak element if  $A[i - 1] \leq A[i] \geq A[i + 1]$ . To simplify the definition of peak elements on the rims of  $A$ , we introduce *sentinel-elements*  $A[0] = A[n + 1] = -\infty$ .

- (a) Give an algorithm with runtime  $\mathcal{O}(\log n)$  (measured in the number of read operations on the array) which returns the position  $i$  of a peak element.
- (b) Prove that your algorithm always returns a peak element, give a recurrence relation for the runtime and use it to prove the runtime.