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Algorithms and Data Structures
Summer Term 2019
Sample Solution Exercise Sheet 2

Exercise 1: $\mathcal{O}$-Notation

Proof or disprove the following statements:

(a) $n \in \Theta(\log_2 3^n)$

(b) $2n \in \mathcal{O}(10\sqrt{n})$

(c) $8n^3 + 5n^2 \in \mathcal{O}(\frac{n^3}{2})$

Sample Solution

(a) The statement is true. Choose $c = \log_2 3$ und $n_0 = 1$. Then for all $n \geq n_0$ we have

$$\log_2 3^n = n \cdot \log_2 3 = c \cdot n.$$

(b) The statement is false. Let $c > 0$. We have

$$\begin{aligned} & 2n \leq c \cdot 10\sqrt{n} \\ \Leftrightarrow \quad & n \leq c \cdot 5\sqrt{n} \\ \Leftrightarrow \quad & \sqrt{n} \leq 5c \\ \Leftrightarrow \quad & n \leq 25c^2 \end{aligned}$$

So for every $c > 0$ and n_0 there is an $n \geq n_0$ with $2n > c \cdot 10\sqrt{n}$ (e.g., $n = \max\{n_0, \lceil 25c^2 \rceil\}$).

(c) The statement is true. For all $n \geq 1$ we have $n^2 \leq n^3$ and thus $8n^3 + 5n^2 \leq 13n^3 = 26\frac{n^3}{2}$.

Exercise 2: Sort Functions by Asymptotic Growth

Sort the following functions by asymptotic growth using the $\mathcal{O}$-notation. Write $g <_{\mathcal{O}} f$ if $g \in \mathcal{O}(f)$ *and* $f \notin \mathcal{O}(g)$. Write $g =_{\mathcal{O}} f$ if $f \in \mathcal{O}(g)$ *and* $g \in \mathcal{O}(f)$.

n^2	$\sqrt{n}$	2^n	$\log(n^2)$
3^n	n^{100}	$\log(\sqrt{n})$	$(\log n)^2$
$\log n$	$10^{100}n$	$n!$	$n \log n$
$n \cdot 2^n$	n^n	$\sqrt{\log n}$	n

Sample Solution

$$
\begin{array}{lllllll}
 & \sqrt{\log n} & <_{\mathcal{O}} & \log(\sqrt{n}) & =_{\mathcal{O}} & \log n & =_{\mathcal{O}} & \log(n^2) \\
<_{\mathcal{O}} & (\log n)^2 & <_{\mathcal{O}} & \sqrt{n} & <_{\mathcal{O}} & n & =_{\mathcal{O}} & 10^{100} n \\
<_{\mathcal{O}} & n \log n & <_{\mathcal{O}} & n^2 & <_{\mathcal{O}} & n^{100} & <_{\mathcal{O}} & 2^n \\
<_{\mathcal{O}} & n \cdot 2^n & <_{\mathcal{O}} & 3^n & <_{\mathcal{O}} & n! & <_{\mathcal{O}} & n^n
\end{array}
$$