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Dept. of Computer Science
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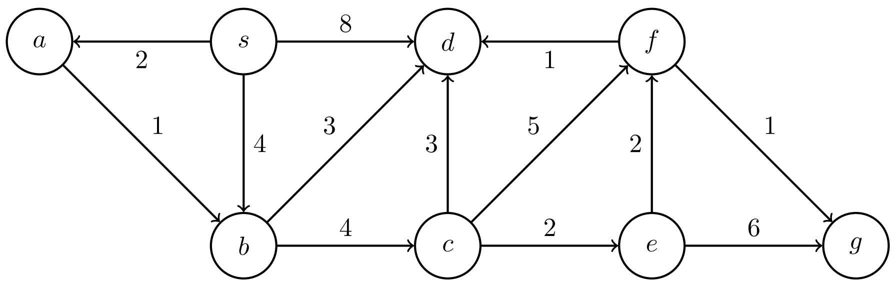
Algorithms and Data Structures
Summer Term 2021
Exercise Sheet 10

Exercise 1: Dijkstras' Algorithm

Execute Dijkstras' Algorithm on the following weighted, directed graph, starting at node s. Into the table further below, write the distances from each node to s that the algorithm stores in the priority queue after each iteration.



Initialization	s	a	b	c	d	e	f	g
$\delta(s,\cdot) =$	0	∞	∞	∞	∞	∞	∞	∞
1. Step $(u = s)$	s	a	b	c	d	e	f	g
$\boldsymbol{\delta(s,\cdot) =}$								
2. Step $(\boldsymbol{u =} \quad)$	s	a	b	c	d	e	f	g
$\boldsymbol{\delta(s,\cdot) =}$								
3. Step $(\boldsymbol{u =} \quad)$	s	a	b	c	d	e	f	g
$\boldsymbol{\delta(s,\cdot) =}$								
4. Step $(\boldsymbol{u =} \quad)$	s	a	b	c	d	e	f	g
$\boldsymbol{\delta(s,\cdot) =}$								
5. Step $(\boldsymbol{u =} \quad)$	s	a	b	c	d	e	f	g
$\boldsymbol{\delta(s,\cdot) =}$								
6. Step $(\boldsymbol{u =} \quad)$	s	a	b	c	d	e	f	g
$\boldsymbol{\delta(s,\cdot) =}$								
7. Step $(\boldsymbol{u =} \quad)$	s	a	b	c	d	e	f	g
$\boldsymbol{\delta(s,\cdot) =}$								
8. Step $(\boldsymbol{u =} \quad)$	s	a	b	c	d	e	f	g
$\boldsymbol{\delta(s,\cdot) =}$								

Exercise 2: Currency Exchange

Consider n currencies $w_1, \dots, w_n$. The exchange rates are given in an $n \times n$-matrix A with entries a_{ij} $(i, j \in \{1, \dots, n\})$. Entry a_{ij} is the exchange rate from w_i to w_j, i.e., for one unit of w_i one gets a_{ij} units of w_j.

Given a currency w_{i_0}, we want to find out whether there is a sequence $i_0, i_1, \ldots, i_k$ such that we make profit if we exchange one unit of w_{i_0} to w_{i_1}, then to w_{i_2} etc. until w_{i_k} and then back to w_{i_0}.

(a) Translate this problem to a graph problem. That is, define a graph and a property which the graph fulfills if and only if there is a sequence of currencies as described above.

(b) Give an algorithm that decides in $\mathcal{O}(n^3)$ time steps whether there is a sequence of currencies as described above. Explain the correctness and runtime.

Hint: $\log(a \cdot b) = \log a + \log b$.