

# Algorithms and Data Structures

## Summer Term 2021

### Exercise Sheet 11

#### Exercise 1: Bitstrings without consecutive ones

Given a positive integer  $n$ , we want to compute the number of  $n$ -digit bitstrings without consecutive ones (e.g., for  $n = 3$  this number is 5, as 000, 001, 010, 100, 101 are the 3-digit bitstrings without consecutive ones).

- (a) Give an algorithm which solves this problem in time  $\mathcal{O}(n)$ . Explain the runtime.
- (b) Implement your solution. You may use the template `DP.py`.

#### Exercise 2: Partitioning

Given a set  $X = \{x_0, \dots, x_{n-1}\}$  with  $x_i \in \mathbb{N}$ , we want to determine whether *there is* a subset  $S \subseteq X$  such that  $\sum_{x \in S} x = \sum_{x \in X \setminus S} x$  (it is not necessary to compute  $S$ ).

- (a) Let  $W := \sum_{x \in X} x$ . Give a recursive formula  $s : \{0, \dots, n-1\} \times \{0, \dots, W\} \rightarrow \{\text{True}, \text{False}\}$  such that  $s(i, j) = \text{True}$  if and only if there is a  $S \subseteq \{x_0, \dots, x_i\}$  such that  $\sum_{x \in S} x = j$ . Explain how  $s$  can be used to solve the above problem in time  $\mathcal{O}(W \cdot n)$ .
- (b) Implement your solution. You may use the template `DP.py`. Run your algorithm on the sets given in `set1.txt`, `set2.txt` and `set3.txt`.