



Theory of Distributed Systems

Exercise Sheet 1

Due: Wednesday, 28th of April 2021, 12:00 noon

Exercise 1: Schedules

(5 Points)

Consider three nodes, v_1 , v_2 , and v_3 , which are connected via FIFO channels, that is, messages between any two nodes are received in the same order they are sent. For example, if node v_1 sends first message m_1 then m_2 to node v_2 , then v_2 will first receive m_1 and then m_2 .

Devise **one** possible schedule S which is consistent with the following local restrictions to the three nodes.

- $S|1 = s_{1,3} s_{1,3} r_{1,2} r_{1,3} s_{1,2} r_{1,2} s_{1,3}$,
- $S|2 = s_{2,3} s_{2,1} r_{2,1} s_{2,1}$,
- $S|3 = r_{3,2} r_{3,1} s_{3,1} r_{3,1} r_{3,1}$.

$s_{i,j}$ denotes the send event from node i to node j and $r_{j,i}$ denotes the event that node j receives a message from node i .

Exercise 2: The Level Algorithm

(5 Points)

Consider the following algorithm between two connected nodes u and v :

The two nodes maintain levels ℓ_u and ℓ_v , which are both initialized to 0. One round of the algorithm works as follows:

1. Both nodes send their current level to each other
2. If u receives level ℓ_v from v , u updates its level to $\ell_u := \max\{\ell_u, \ell_v + 1\}$. If the message to node u is lost, node u does not change its level ℓ_u . Node v updates its level ℓ_v in the same (symmetric) way.

Argue that if the level algorithm runs for r rounds, the following properties hold:

- (a) At the end, the two levels differ by at most one.
- (b) If all messages succeed, both levels are equal to r .
- (c) The level of a node is at least 1 if and only if the node received at least one message.

Exercise 3: (Variations) of Two Generals

(10 Points)

In the lecture we considered the (deterministically unsolvable) **Two Generals** consensus problem:

- two deterministic nodes, synchronous communication, unreliable messages,
- **input**: 0 or 1 for each node,

- **output:** each node needs to decide either 0 or 1,
- **agreement:** both nodes must output the same decision (0 or 1),
- **validity:** if both nodes have the same input $x \in \{0, 1\}$ and no messages are lost, both nodes output x ,
- **termination:** both nodes terminate in a bounded number of rounds.

In this exercise we consider three modifications of the model. For each of them, either give a (deterministic) algorithm or state a proof which shows that the variation cannot be solved deterministically.

- (a) There is the guarantee that within the first 10 rounds at least *one* message in *each* direction succeeds.
- (b) There is the guarantee that within the first 10 rounds at least *one* message succeeds
- (c) Let $k \in \mathbb{N}$ be a natural number. The input for each node is a number $x \in \{0, \dots, k\}$.

Goal: If no message gets lost *and* both have the same input $x \in \{0, \dots, k\}$, both have to output x . In all other cases the nodes should output numbers which do not differ by more than one. The algorithm still has to terminate in a finite number of rounds.