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Theory of Distributed Systems
Exercise Sheet 12

Due: Wednesday, 23rd of July 2021, 12:00 noon

Exercise 1: Aggregation in the MPC Model *(15 Points)*

Assume you are given a number of $M \in O\big(\frac{N}{S}\log_S N\big)$ machines, where N is the number of *aggregation messages* that are collectively stored by the machines M_i, $i \in \{1, \dots, M\}$. Each machine M_i has a memory large enough to store S such messages. By definition of the MPC model every machine can send and receive at most S aggregation messages per round.

Each aggregation message m has an aggregation value v_m, a target machine t_m and an aggregation group g_m. All messages in the same group go to the same target and each machine is the target of not more than one aggregation group. The *aggregation problem* is solved when every target machine t_m learns an aggregation message m that has *minimal* value among all aggregation messages of its aggregation group g_m. Formulate an algorithm that solves said aggregation problem in $O(\log_S N)$ rounds such that no machine sends or receives more than $S/2$ messages per round *in expectation.*

Simplifications: You may assume that the intial aggregation messages are stored on $\lceil \frac{N}{S} \rceil$ machines and none of those machines is a target of an aggregation message. This means that machines can be partitioned into $O(\log_S N)$ levels with a separate level for sources and targets of aggregation messages, respectively. You may further assume that we have sufficient long string of "public random bits", which can be used to make random decisions that are the same for all machines, (since all machines utilize the same random bit string).

Exercise 2: Implement a Phase of Borůvka's Algorithm *(5 Points)*

In class, we sketched how to implement one phase of Borůvka's MST algorithm in the strongly sublinear regime $S = n^{\alpha}$ for some constant $0 < \alpha < 1$. Argue in more detail how this can be done in $O(1)$ rounds, given that we can solve the above aggregation problem.