Exercise 1: Red-Black Trees

(a) Decide for each of the following trees if it is a red-black tree and if not, which property is violated:

(b) On the following red-black tree, first execute the operation `insert(8)` and afterwards `delete(5)`. Draw the resulting tree and document intermediate steps.

Exercise 2: AVL-Trees

An AVL-tree is a binary search tree with the additional property that for each node $v$, the depth of its left and its right subtree differ by at most 1.

(a) Show via induction that an AVL-tree of height $d$ is filled completely up to depth $\lfloor \frac{d}{2} \rfloor$. (3 Points)

A binary tree is filled completely up to depth $d'$ if it contains for all $x \leq d'$ exactly $2^x$ nodes of depth $x$.

1AVL-trees were not part of the lecture. For this task you only need the definition given below.
(b) Give a recursion relation that describes the minimum number of nodes of an AVL-tree as a function of $d$. (3 Points)

(c) Show that an AVL-tree with $n$ nodes has depth $O(\log n)$. (4 Points)

You can either use part (a) or part (b).