Chapter 4
Causality, Time, and Global States

Distributed Systems

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Time in Distributed Systems

Goal: Establish a notion of time in (partially) asynchronous systems

Physical time:

- Establish an approximation of real time in a network
- Synchronize local clocks in a network
- Timestamp events (email, sensor data, file access times etc.)
- Synchronize audio and video streams
- Measure signal propagation delays (Localization)
- Wireless (TDMA, duty cycling)
- Digital control systems (ESP, airplane autopilot etc.)

Logical time:

- Determine an order on the events in a distributed system
- Establish a global view on the system

Logical Clocks

Goal: Assign a timestamp to all events in an asynchronous message-passing system

- Allows to give the nodes some notion of time
  - which can be used by algorithms
- **Logical clock values:** numerical values that increase over time and which are consistent with the observable behavior of the system
- The objective here is **not** to do **clock synchronization**:

  Clock Synchronization: compute logical clocks at all nodes which simulate real time and which are tightly synchronized.
  - We might talk about clock synchronization later...

Observable Behavior

Recall Executions / Schedules

- An exec. is an alternating sequence of configurations and events
- A schedule S is the sequence of events of an execution
  - Possibly including node inputs
- Schedule restriction for node v:

$$S|v \coloneqq \text{"sequence of events seen by } v\text{"}$$

Causal Shuffles

We say that a schedule $\boldsymbol{S'}$ is a **causal shuffle** of schedule $\boldsymbol{S}$ iff

$$\forall \boldsymbol{v} \in \boldsymbol{V}: \quad \boldsymbol{S}|\boldsymbol{v} = \boldsymbol{S'}|\boldsymbol{v}.$$

Observation: If S' is a causal shuffle of S, no node/process can distinguish between S and S'.

Causal Order

Logical clocks are based on a **causal order** of the events

- In the order, event e should occur before event e' if event e provably occurs before event e'
  - In that case, the clock value of e should be smaller than the one of e'

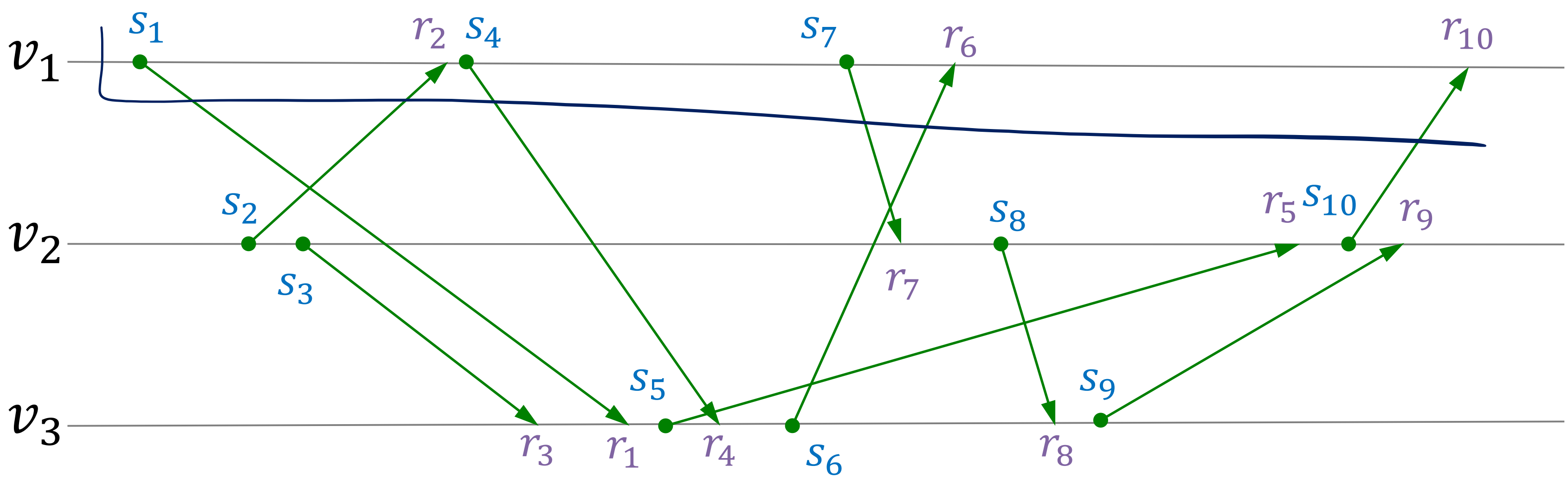
For a given schedule S:

- The distributed system cannot distinguish S from another schedule S' if and only if S' is a causal shuffle of S.
  - causal shuffle $\Longrightarrow$ no node can distinguish
  - no causal shuffle $\Longrightarrow$ some node can distinguish

Event e provably occurs before e' if and only if e appears before e' in all causal shuffles of S

Causal Shuffles / Causal Order Example

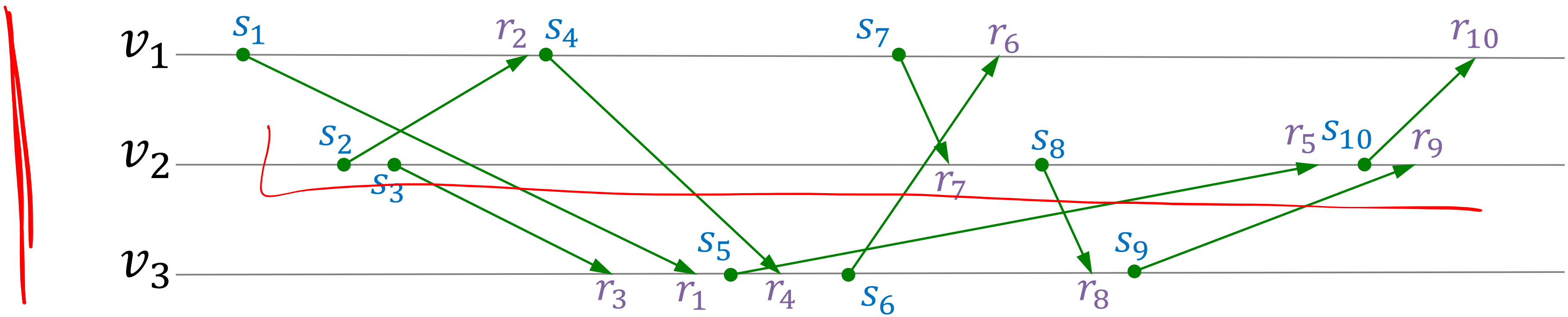
Schedule S $= s_1\ s_2\ s_3\ r_2\ s_4\ r_3\ r_1\ \ldots$



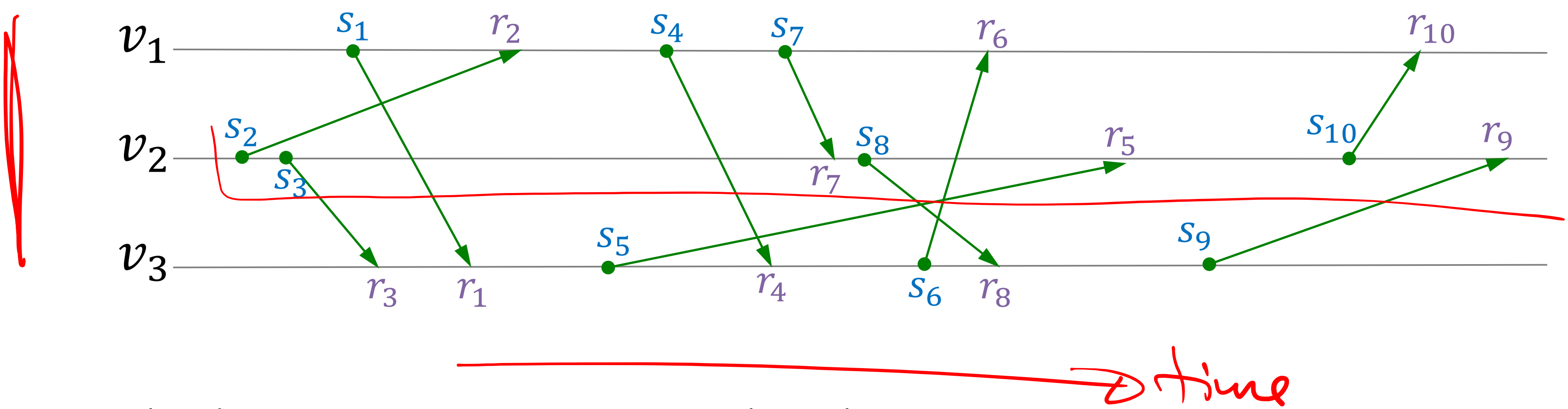
$S|v_1 = s_1, r_2, s_4, s_7$

Causal Shuffles / Causal Order Example

Schedule S s_1, s_2, s_3, r_2, r_4



Some Causal Shuffle S' $s_2, s_3, s_1, r_3, r_1, r_2$



Lamport's Happens-Before Relation

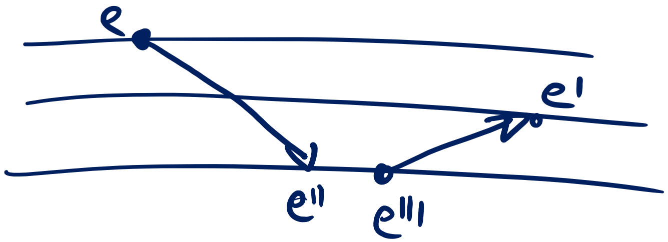
Assumption: message passing system, only send and receive events

Consider two events e and e' occurring at nodes u and u'

- send event occurs at sending node, recv. event at receiving node
- let's define t and t' be the (real) times when e and e' occur

We know that e provably occurs before e' if

1. The events occur at the same node and e occurs before e'
2. Event e is a send event, e' the recv. event of the same message
3. There is an event e'' for which we know that provably, e occurs before e'' and e'' occurs before e'



Lamport's Happens-Before Relation

Definition: The **happens-before relation** $\Rightarrow_S$ on a schedule S is a pairwise relation on the send/receive events of S and it contains

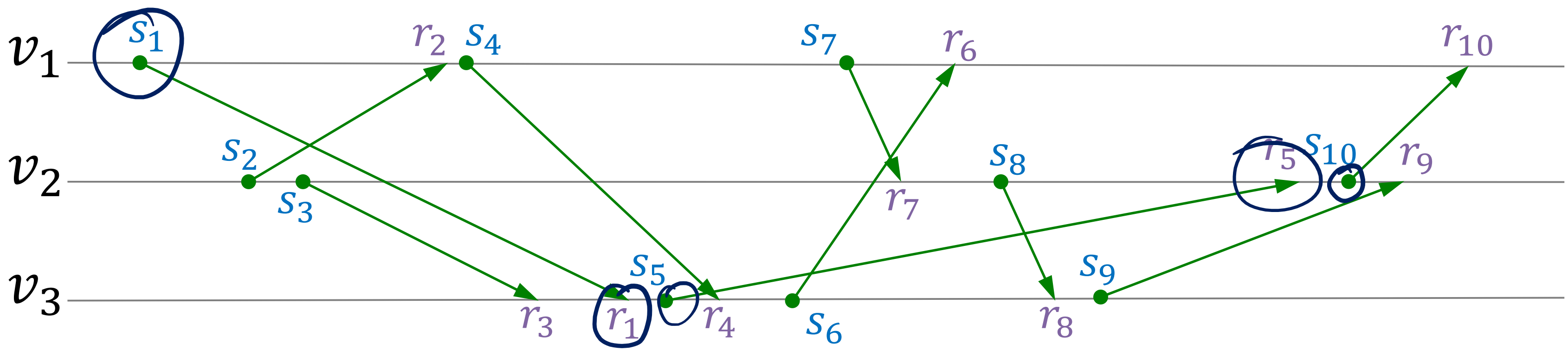
1. All pairs (e, e') where e precedes e' in S and e and e' are events of the same node/process.
2. All pairs (e, e') where e is a send event and e' the receive event for the same message.
3. All pairs (e, e') where there is a third event e'' such that

$$e \Rightarrow_S e'' \quad \land \quad e'' \Rightarrow_S e'$$

   – Hence, we take the **transitive closure** of the relation defined by 1. and 2.

Happens-Before Relation: Example

Schedule S



$s_1 \Rightarrow_S r_1 \Rightarrow_S s_5 \Rightarrow_S r_5 \Rightarrow_S s_{10}$

$\Downarrow$

$s_1 \Rightarrow_S s_{10}$

Happens-Before and Causal Shuffles

Theorem: For a schedule S and two (send and/or receive) events e and e', the following two statements are equivalent:

a) Event e happens-before e', i.e., $\boldsymbol{e \Rightarrow_S e'}$.

b) Event e precedes e' in all causal shuffles S' of S.

Some remarks before proving the theorem…

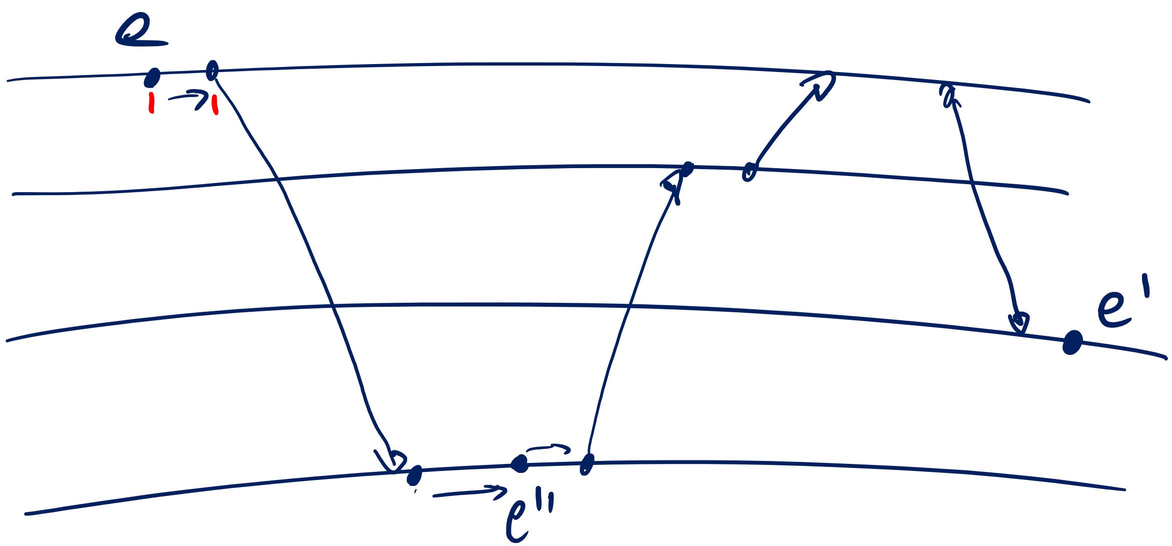
- Shows that the happens-before relation is exactly capturing what we need about the causality between events
  - It captures exactly what is observable about the order of events

- To prove the theorem, we show that
  1. a) $\longrightarrow$ b)
  2. b) $\longrightarrow$ a)

Happens-Before and Causal Shuffles

If $e \Rightarrow_S e'$, then e precedes e' in all causal shuffles S' of S.

- e, e' are on same node ✓
- e, e' belong to the same msg. (e: send, e': rcv.) ✓
-

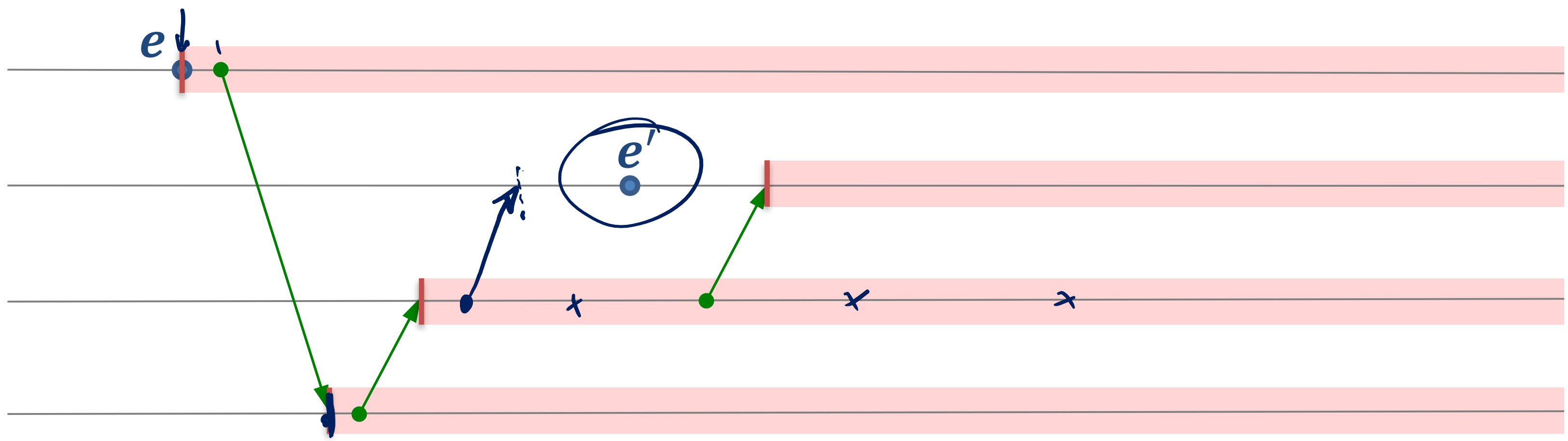


Happens-Before and Causal Shuffles

If e precedes e' in all causal shuffles S' of S, then $e \Rightarrow_S e'$.

Proof: $\neg(e \Rightarrow_S e')$ $A \Rightarrow B$ $\neg B \to \neg A$

- Show: $e \not\Rightarrow_S e'$, there is a shuffle S' such that e' precedes e in S
- W.l.o.g., assume that e precedes e' in S
  - Consequently, e and e' happen at different nodes (otherwise, the order remains the same in all causal shuffles)



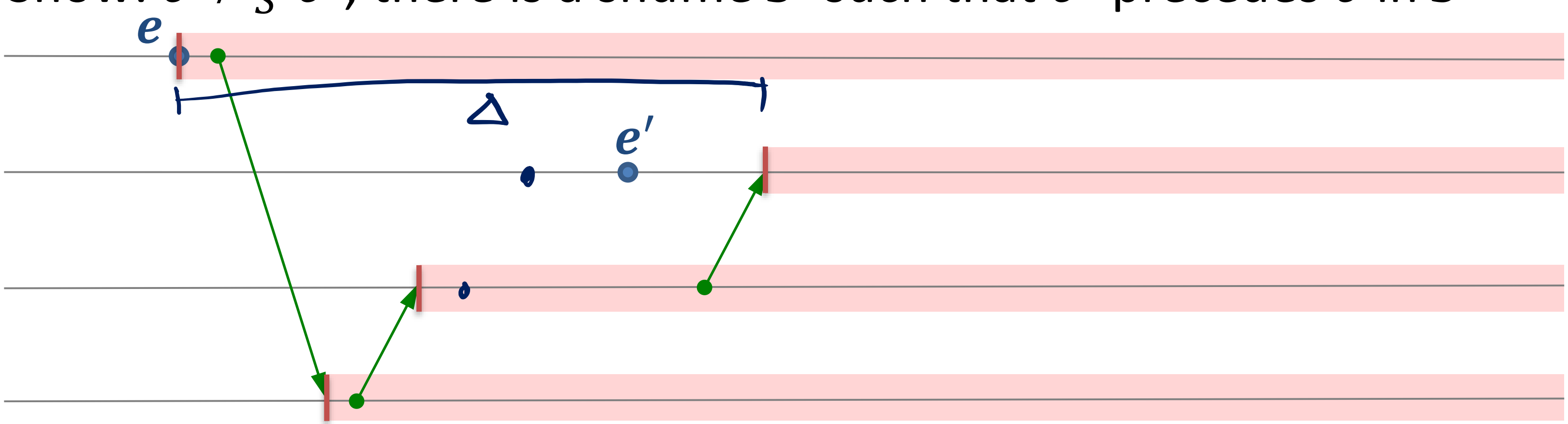
- **Events in red part can be shifted by fixed amount Δ**

Happens-Before and Causal Shuffles

If e precedes e' in all causal shuffles S' of S, then $e \Rightarrow_S e'$.

Proof:

- Show: $e \not\Rightarrow_S e'$, there is a shuffle S' such that e' precedes e in S



- **Events in red part can be shifted by fixed amount Δ**
  - Consider some message M with send/receive events s_M, r_M
  - If s_M and r_M or only r_M are shifted, message delay gets larger → OK
  - It is not possible to only shift s_M
  - Choose Δ large enough to move e past e'

Lamport Clocks

Basic Idea:

1. Each event e gets a clock value $\tau(e) \in \mathbb{N}$
2. If e and e' are events at the same node and e precedes e', then
$$\tau(e) < \tau(e')$$
3. If s_M and r_M are the send and receive events of some msg. M,
$$\tau(s_M) < \tau(r_M)$$

Observation:

- For clock values $\tau(e)$ of events e satisfying 1., 2., and 3., we have
$$\boldsymbol{e \Rightarrow_S e' \quad \longrightarrow \quad \tau(e) < \tau(e')}$$
  – because $<$ relation (on $\mathbb{N}$) is transitive
- Hence, the partial order defined by $\tau(e)$ is a superset of $\Rightarrow_S$

Lamport Clocks

Algorithm:

- Each node u keeps a counter c_u which is initialized to 0
- For any non-receive event e at node u, node u computes

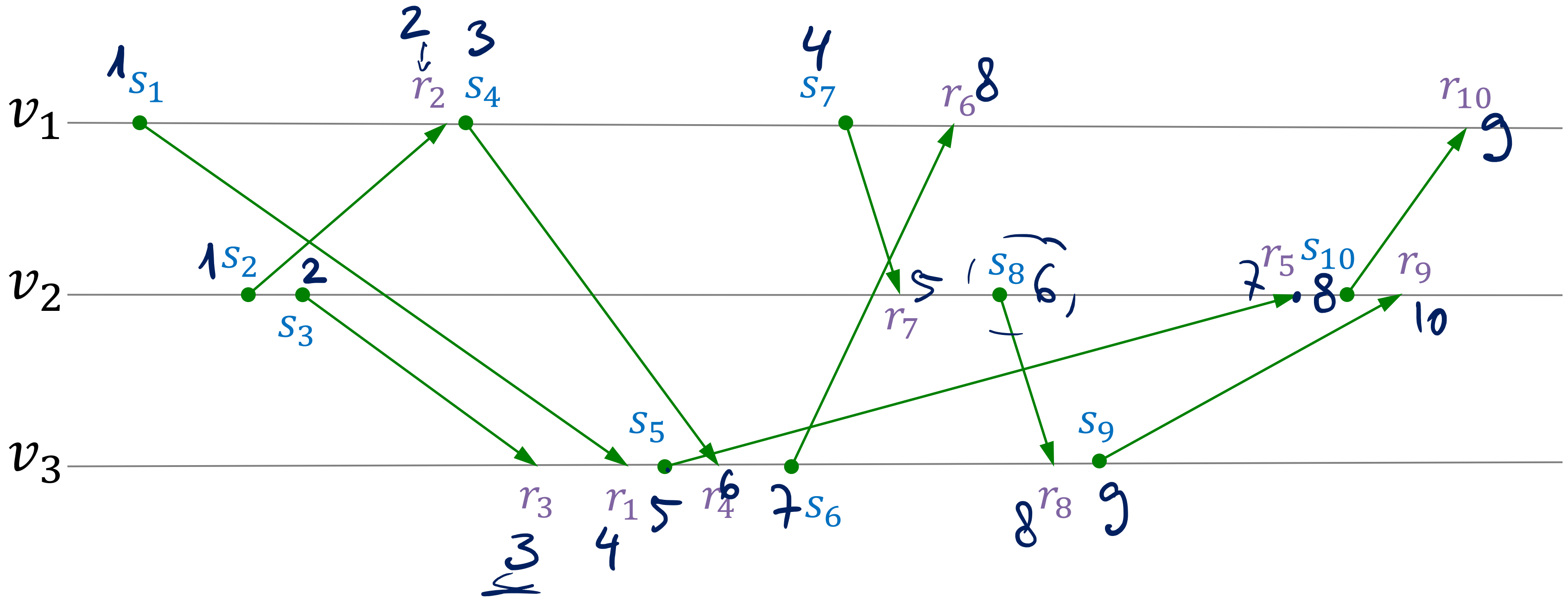
$$c_u \coloneqq c_u + 1;\ \tau(e) \coloneqq c_u$$

- For any send event s at node u, node u attaches the value of $\tau(s)$ to the message
- For any receive event r at node u (with corresponding send event s), node u computes

$$c_u \coloneqq \max\{c_u, \tau(s)\} + 1;\ \ \tau(r) \coloneqq c_u$$

Lamport Clocks: Example

Schedule S



Neiger-Toueg-Welch Clocks

Discussion Lamport Clocks:

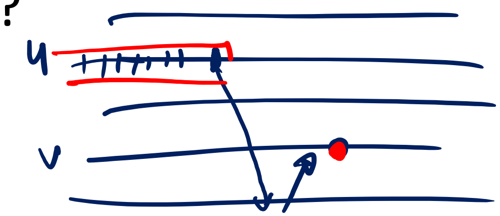
- Advantage: no changes in the behavior of the underlying protocol
- Disadvantage: clocks might make huge jumps (when recv. a msg.)

Idea by Neiger, Toueg, and Welch:

- Assume nodes have some approximate knowledge of real time
  - e.g., by using a clock synchronization algorithm
- Nodes increase their clock value periodically
- Combine with Lamport clock ideas to ensure safety
- When receiving a message with a time stamp which is larger than the current local clock value, wait with processing the message.

Fidge-Mattern Vector Clocks

- Lamport clocks give a superset of the happens-before relation
- Can we compute logical clocks to get $\Rightarrow_S$ exactly?



Vector Clocks:

- Each node u maintains an vector $\mathrm{VC}(u)$ *of* clock values
  - one entry $\mathrm{VC}_v(u)$ for each node $v \in V$
- In the vector $\mathrm{VC}(e)$ assigned (by u) to some event e happening at node u, the component x_v corresponding to $v \in V$ refers to the

  number of events at node v, u knows about when e occurs

Vector Clocks Algorithm

- All Nodes u keep a vector $\mathrm{VC}(u)$ with an entry for all nodes in V
  - all components are initialized to 0
  - component corresponding to node v: $\mathrm{VC}_v(u)$
- For any non-receive event e at node u, node u computes

$$\mathrm{VC}_u(u) \coloneqq \mathrm{VC}_u(u) + 1; \quad \mathrm{VC}(e) \coloneqq \mathrm{VC}(u)$$

- For any send event s at node u, node u attaches the value of $\mathrm{VC}(s)$ to the message
- For any receive event r at node u (with corresponding send event s), node u computes

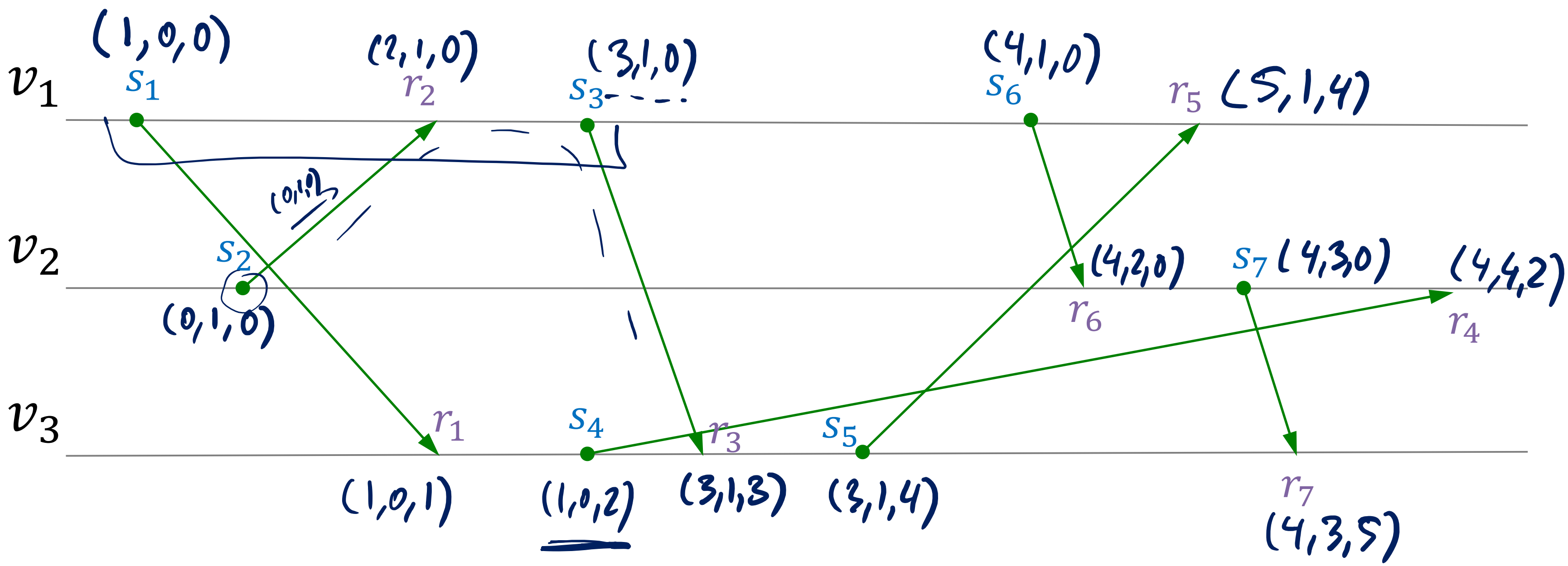
$$\forall v \neq u: \mathrm{VC}_v(u) \coloneqq \max\{\mathrm{VC}_v(s), \mathrm{VC}_v(u)\};$$
$$\mathrm{VC}_u(u) \coloneqq \mathrm{VC}_u(u) + 1;$$
$$\mathrm{VC}(r) \coloneqq \mathrm{VC}(u)$$

Vector Clocks Example

(x, y, z)

v₁ v₂ v₃

Schedule S



Vector Clocks and Happens-Before

Definition: $\mathbf{VC}(\boldsymbol{e}) < \mathbf{VC}(\boldsymbol{e}') :=$

$$\big(\forall \boldsymbol{v} \in \boldsymbol{V}: \underline{\mathbf{VC}_{\boldsymbol{v}}(\boldsymbol{e}) \leq \mathbf{VC}_{\boldsymbol{v}}(\boldsymbol{e}')}\big) \land \big(\mathbf{VC}(\boldsymbol{e}) \neq \boldsymbol{VC}(\boldsymbol{e}')\big)$$

Theorem: Given a schedule S, for any two events e and e',

$$\underline{\mathbf{VC}(\boldsymbol{e}) < \mathbf{VC}(\boldsymbol{e}')} \quad \longleftrightarrow \quad \underline{\boldsymbol{e} \Rightarrow_{\boldsymbol{s}} \boldsymbol{e}'}$$

- see exercises!

Logical Clocks vs. Synchronizers

Synchronizer:

- Algorithm that generates clock pulses that allow to run a synchronous algorithm in an asynchronous network
  - We will discuss synchronizers later

The clock pulses (local round numbers) generated by a synchronizer can also used as logical clocks

- Send events of round r get clock value $2r - 1$
- Receive events of round r get clock value $2r$
- superset of the happens-before relation
- requires to drastically change the protocol and its behavior
  - synchronizer determines when messages can be sent
- a very heavy-weight method to get logical clock values
  - requires a lot of messages

Application of Logical Times

Replicated State Machine

- main application suggested by Lamport in his original paper
- a shared state machine where every node can issue operations
- state machine is simulated by several replicas

Solution:

- add current clock value (and issuer node ID) to every operation
- operations have to be carried out in order of clock values / IDs
- ***Safety:***
  - all replicas use same order of operations
  - order of operations is a possible actual order (consistent with local views)
- ***Liveness:***
  - progress is guaranteed if nodes regularly send messages to each other

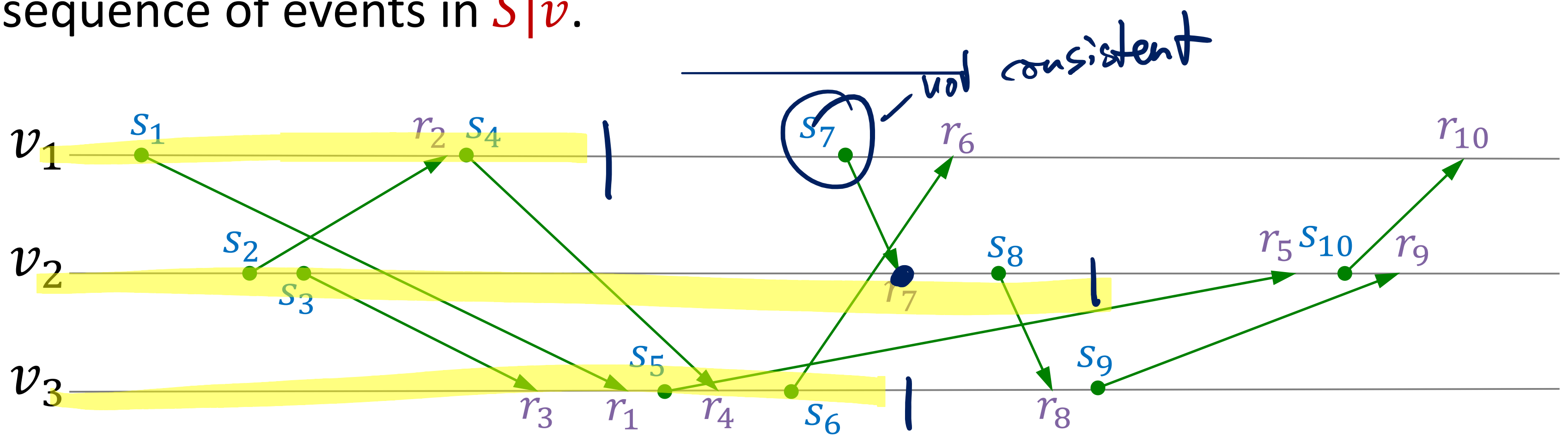
Global States

- Sometimes the nodes of a distributed system need to figure out the global state of the system
  - e.g., to find out if some property about the system state is true
- Executions/schedules which lead to the same happens-before relation (i.e., causal shifts) cannot be distinguished by the system.
- Generally not possible to record the global state at any given time of the execution
- Best solution: A global state which is consistent with all local views
  - i.e., a state which could have been true at some time
- Called a **consistent** or **global snapshot** of the system and based on **consistent cuts** of the schedule

Consistent Cut

Cut

Given a schedule S, a cut is a subset C of the events of S such that for all nodes $v \in V$, the events in C happening at v form a prefix of the sequence of events in $S|v$.

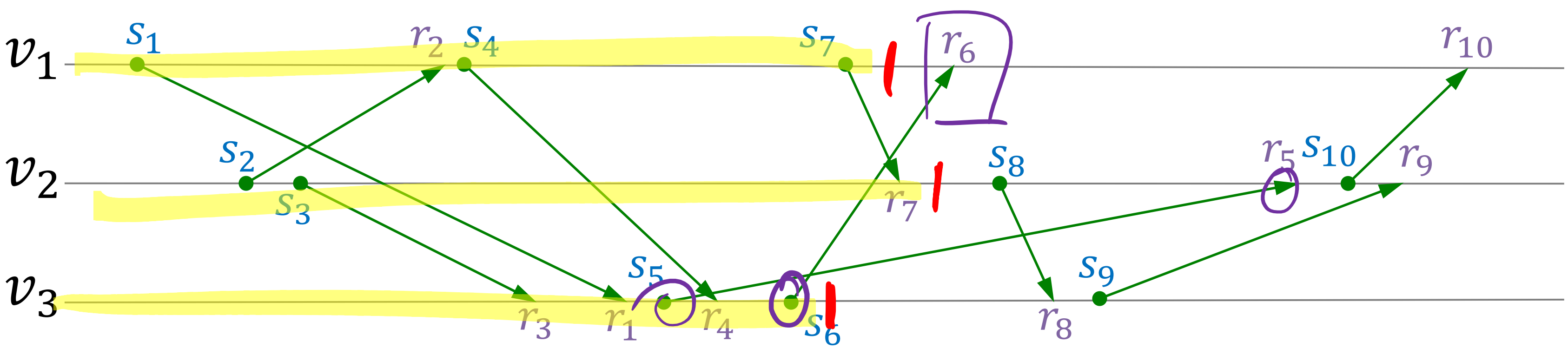


Consistent Cut

Consistent Cut

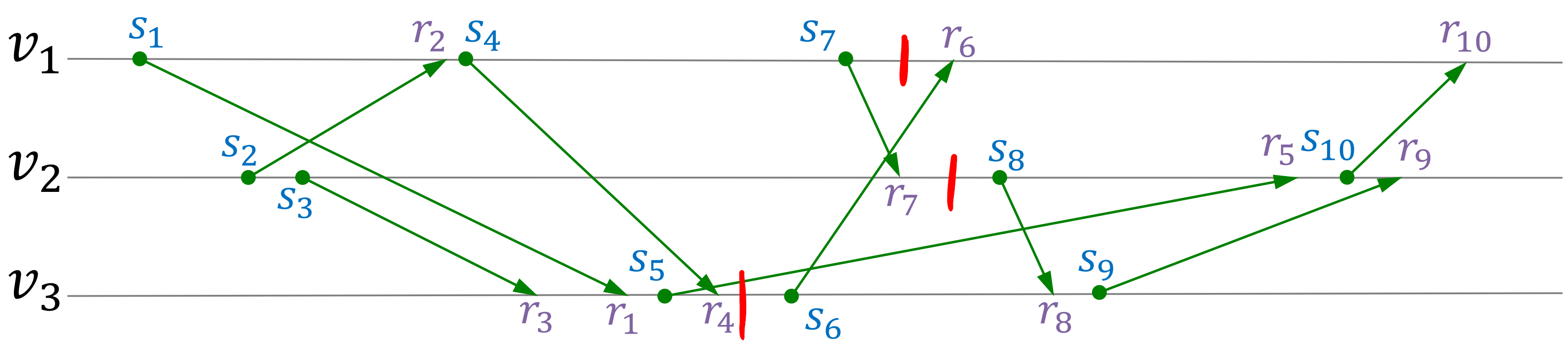
Given a schedule S, a consistent cut C is cut such that for all events $e \in C$ and all events f in S, it holds that

$$\boldsymbol{f \Rightarrow_S e \quad \longrightarrow \quad f \in C}$$

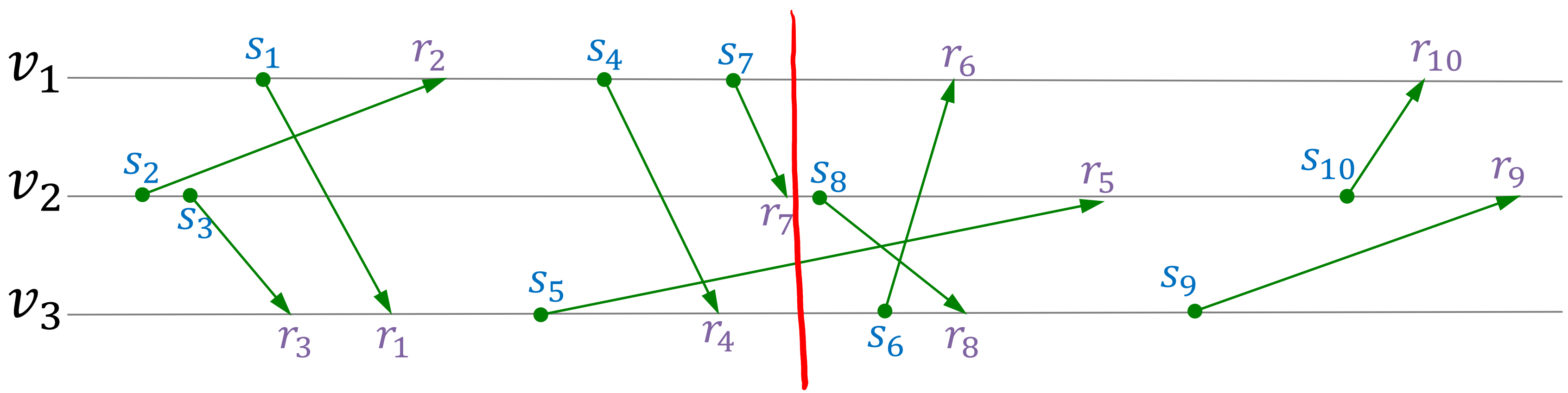


Consistent Cut

Schedule S



Some Causal Shuffle S'



Consistent Cuts

Claim: Given a schedule S, a cut C is a consistent cut if and only if for each message M with send event s_M and receive event r_M, if $r_M \in C$, then it also holds that $s_M \in C$.

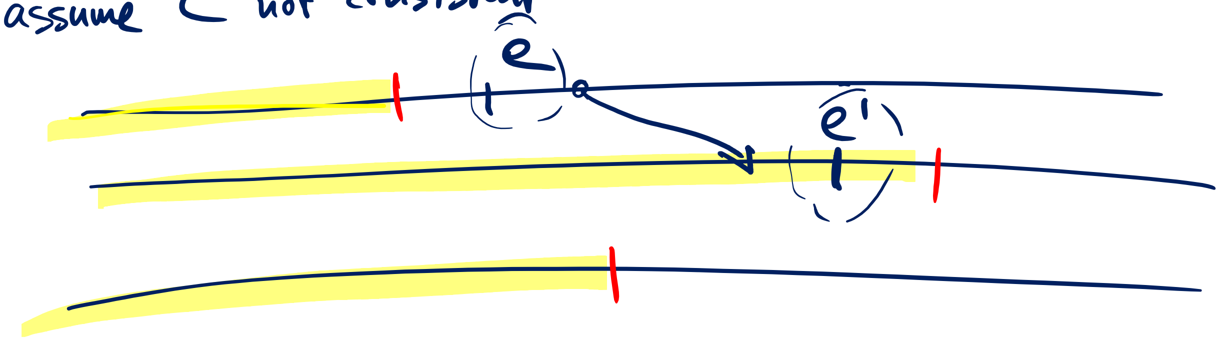
$\forall M: \quad r_M \in C \rightarrow s_M \in C \iff C$ consistent

$\Leftarrow$ by def. of consistent

other dir $\rightarrow$

assume C not consistent

$e \Rightarrow_S e'$



Consistent Snapshot

Consistent Snapshot = Global Snapshot = Consistent Global State

- A consistent snapshot is a global system state which is consistent with all local views.

Global System State (for schedule S)

- A vector of intermediate states (in S) of all nodes and a description of the messages currently in transit
  - Remark: If nodes keep logs of messages sent and received, the local states contain the information about messages in transit.

Consistent Snapshot

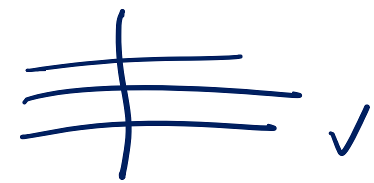
- A global system state which is an intermediate global state for some causal shuffle of S (consistent with all local views)

Consistent Snapshot

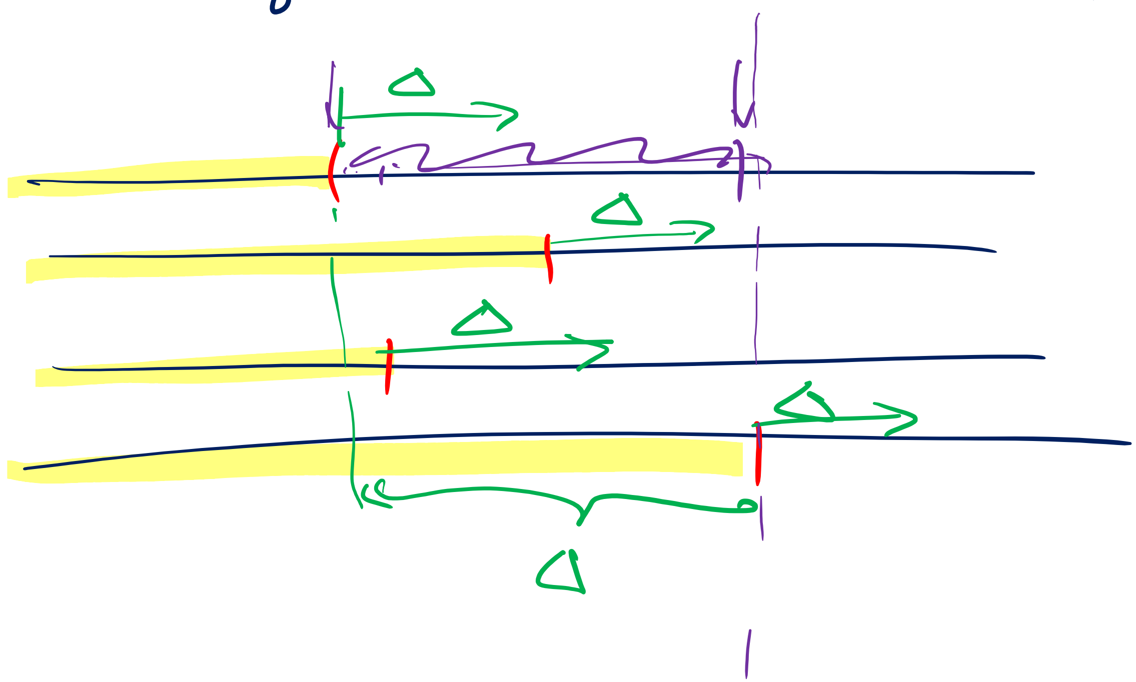
Claim: A global system state is a consistent snapshot if and only if it corresponds to the node states of some consistent cut C.

cons. snapshot $\underset{}{\overset{1}{\rightleftarrows}}$ cons. cut

1) intermediate global state in some causal shuffle



2)



Computing a Consistent Snapshot

Using Logical Clocks

- Assume that each event e has a clock value $\tau(e)$ such that for two events e, e',

$$e \Rightarrow_S e' \quad \longrightarrow \quad \tau(e) < \tau(e')$$

- Given τ, define $C(\tau)$ as the set of events e with $\tau(e) \leq \tau_0$

Claim: $\forall \tau \geq 0$: $C(\tau)$ is a consistent cut.

Remark: Not always clear how to choose τ

- τ large: not clear how long it takes until snapshot is computed
- τ small: snapshot is "less up-to-date"

Chandy-Lamport Snapshot Algorithm

Goals: Compute a consistent snapshot in a running system

Assumptions:

- Does not require logical clocks
- Channels are assumed to have FIFO property
- No failures
- Network is (strongly) connected
- Any node can issue a new snapshot

Remark: The FIFO property can always be guaranteed

  – sender locally numbers messages on each outgoing channel
  – messages with smaller numbers have to be processed before messages with larger numbers
  – works as long as messages are not lost

Chandy-Lamport Snapshot Algorithm

Overview:

- Assume that node s initiates the snapshot computation
- The times for recording the state at different nodes is determined by sending around *marker* messages
- When receiving the first *marker* message, a node records its state and sends *marker* messages to all (outgoing) neighbors
- On each incoming channel, the set of messages which are received between recording the state and receiving the *marker* message (on this channel) are in transit in the snapshot.
- After receiving a *marker* message on all incoming channels, a nodes has finished its part of the snapshot computation

Chandy-Lamport Snapshot Algorithm

Initially: Node s records its state

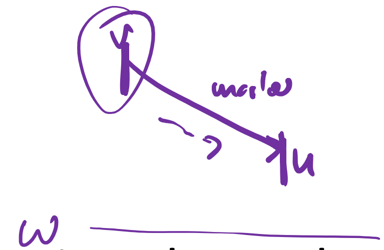
When node u receives a *marker* message from node v:

if u has not recorded its state then

  u records its state

  set of msg. in transit from v to u is empty

  u starts recording messages on all other incoming channels



else

  the set of msg. in transit from v to u is the set of recorded msg.

  since starting to record msg. on the channel

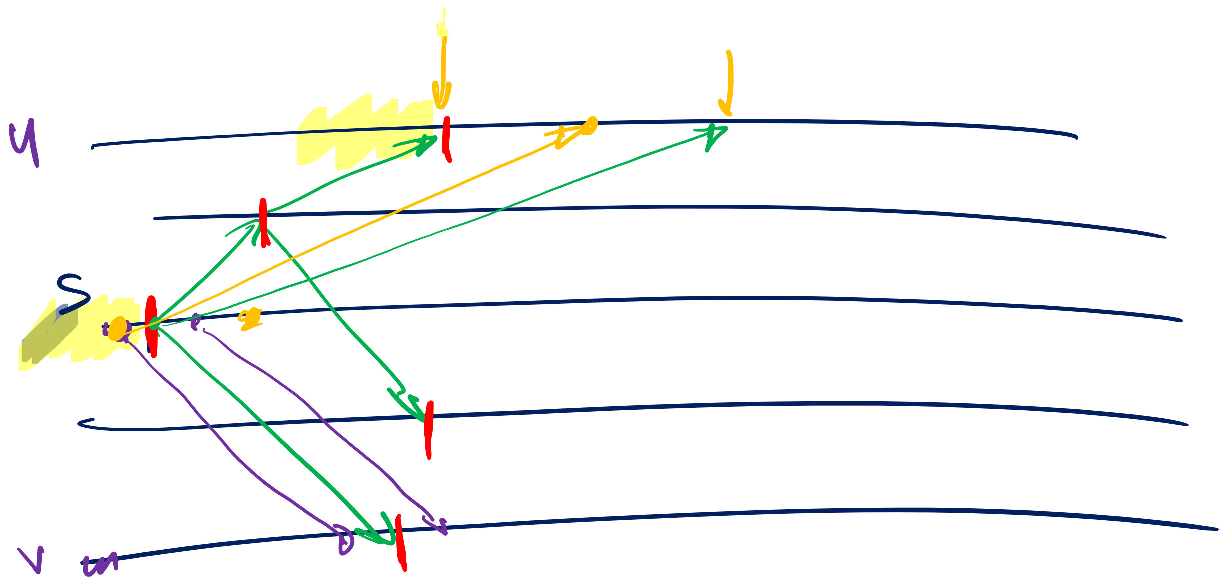
(Immediately) after node u records its state:

Node u sends *marker* msg. on all outgoing channels

- before sending any other message on those channels

Chandy-Lamport Snapshot Algorithm

Theorem: The Chandy-Lamport algorithm computes a consistent cut and it correctly computes the messages in transit over this cut.



Chandy-Lamport Snapshot Algorithm

Theorem: The Chandy-Lamport algorithm computes a consistent cut and it correctly computes the messages in transit over this cut.

Applications of Consistent Snapshots

Testing Stable System Properties

- A stable property is a property which once true, remains true
- More formally: a predicate P on global configurations such that if P is true for some configuration C, P also holds for all configurations which can be reached from C

Testing a stable property:

- test whether property holds for a consistent snapshot

Safety: Only evaluates to true if the property holds

- the current state is reachable from every consistent snapshot state

Liveness: If the property holds, it will eventually be detected

- initiating a snapshot (using Chandy-Lamport) leads to snapshot configuration which is reachable from the current configuration

Applications of Consistent Snapshots

Distributed Garbage Collection

- Erase objects (e.g., variables stored at some node(s)) to which no reference exists any more
- References can be at other nodes, in messages in transit, ...
- "No reference to object x" is a stable system property

Distributed Deadlock Detection

- Two processes/nodes wait for each other
- Deadlock is also a stable property

Distributed Termination Detection

- "Distributed computation has terminated" is a stable property
- Note, need also see messages in transit