



Algorithms and Datastructures

Summer Term 2024

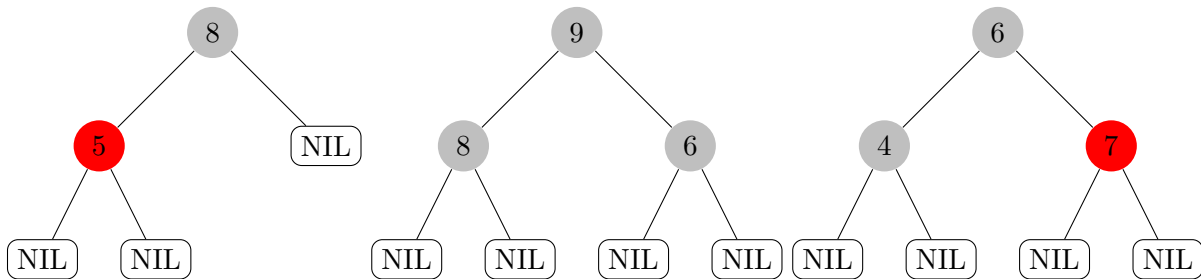
Exercise Sheet 7

Due: Wednesday, June 12th, 2pm

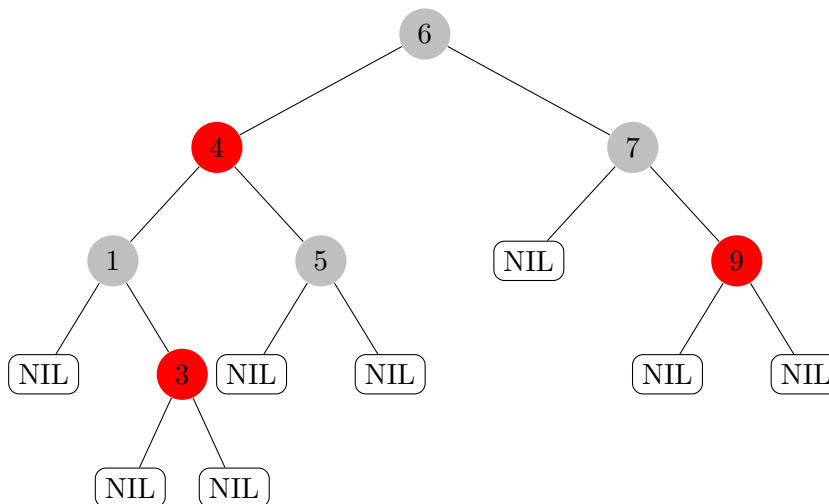
Exercise 1: Red-Black Trees

(10 Points)

(a) Decide for each of the following trees if it is a red-black tree and if not, which property is violated:



(b) On the following red-black tree, first execute the operation `insert(8)` and afterwards `delete(5)`. Draw the resulting tree and document intermediate steps.



Exercise 2: AVL-Trees ¹

(10 Points)

An AVL-tree is a binary search tree with the additional property that for each node v , the depth of its left and its right subtree differ by at most 1.

(a) Show via induction that an AVL-tree of depth d is filled completely up to depth $\lfloor \frac{d}{2} \rfloor$. (3 Points)

A binary tree is filled completely up to depth d' if it contains for all $x \leq d'$ exactly 2^x nodes of depth x .

¹AVL-Trees are not part of the lecture. To solve this exercise the definition given below is sufficient.

(b) Give a recursion relation that describes the minimum number of nodes of an AVL-tree as a function of d . *(3 Points)*

(c) Show that an AVL-tree with n nodes has depth $\mathcal{O}(\log n)$. *(4 Points)*

You can either use part (a) or part (b).