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Theory of Distributed Systems
Exercise Sheet 8

Due: Wednesday, 19th of June 2024, 12:00 noon

Exercise 1: 2-coloring in paths *(7 Points)*

Show that there is no randomized distributed algorithm that finds a 2-coloring in paths in $o(n)$ rounds with probability at least 0.9. Assume that n is known and IDs are from $\{1, \dots, n\}$.
Hint: use that the principle of locality can be extended to randomized algorithms i.e. for any possible output, nodes with the same view have the same probability of giving that output.

Exercise 2: Independent sets in paths *(9 Points)*

An independent set (IS) is a subset of nodes such that no two neighboring nodes are in the independent set. A maximal independent set (MIS) is an independent set that cannot be extended. Assume that n is known and IDs are from $\{1, \dots, n\}$. Show that, in paths:

1. it is trivial to find some IS in $O(1)$ time with a deterministic distributed algorithm.
2. there exists an IS with at least $n/2$ nodes.
3. it is not possible to find an IS of size at least $n/2$ in $o(n)$ rounds.
4. there is no deterministic distributed algorithm that finds an MIS in $o(\log^* n)$ rounds.

Bonus: a *vertex cover* of a graph is a subset of nodes that includes at least one endpoint of every edge of the graph. Deduce from number 3 above that it is not possible in paths to find a vertex cover of size at most $n/2$ in $o(n)$ rounds.

Exercise 3: Counting *(4 Points)*

Assume we are given a path of size n where nodes know an upper bound on the size of the network in $\{n, \dots, cn + c'\}$ for some constants c, c' (i.e., nodes do not necessarily know the exact value of n but only e.g. a constant approximation). Show that there is no deterministic distributed algorithm that counts the number of nodes in paths in $o(n)$ rounds, where at the end of the algorithm every node needs to output this count. Assume that IDs are from $\{1, \dots, n\}$.