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Theory of Distributed Systems
Exercise Sheet 9

Due: Wednesday, 3th of July 2024, 12:00 noon

Exercise 1: Gather everything in CONGEST *(5 Points)*

We are given a connected graph $G = (V, E)$ and we want each node to know the whole graph. We have seen during the lecture that such problem can be solved in $O(diam(G))$ rounds in the LOCAL model. Show that it is possible to solve this problem in $O(|E|)$ in the CONGEST model.

Exercise 2: APSP – Slow token *(5 Points)*

During the lecture we have seen two animations of the APSP algorithm:

- One where the token is moved slowly (every 2 rounds), and there are no waves that collide.
- One where the token is moved fast (every round), and many waves collide.

Prove that it is indeed true that, if we move the token every two rounds, each node at each round has to propagate at most one wave.

Exercise 3: Bipartite Graph Detection *(5 Points)*

In this exercise we say that in order to detect if the graph $G = (V, E)$ has a certain property in the CONGEST model, all nodes have to output 1 if the graph fulfills that property, otherwise all nodes have to output 0. A graph is called bipartite if its nodes can be partitioned into two sets, i.e., $V = A \cup B, A \cap B = \emptyset$, such that for all edges $\{u, v\} \in E$ we have $u \in A$ and $v \in B$ (or vice versa). Show that it is possible to detect if the graph is *bipartite* or not in $\mathcal{O}(\text{diam}(G))$ rounds.

Exercise 4: Diameter Lower Bound – Refinement *(5 Points)*

In the lecture we have seen that computing the diameter $\text{diam}(G)$ of G in the CONGEST model takes $\Omega(n/\log n)$ rounds in general. Argue that the same lower bound holds even for computing an approximate value $\tilde{D}$ with $\frac{4}{5} \cdot \text{diam}(G) < \tilde{D} \leq \text{diam}(G)$.