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Theoretical Computer Science - Bridging Course
Exercise Sheet 11

Due: Tuesday, 9th of July 2024, 12:00 pm

Exercise 1: Construct Formulae *(1+1+1 Points)*

Let $\mathcal{S} = \langle \{x, y, z\}, \emptyset, \emptyset, \{R\} \rangle$ be a signature. Translate the following sentences of first order formula over $\mathcal{S}$ into idiomatic English. Use $R(x, y)$ as statement 'x is a part of y'.

(a) $\exists x \forall y R(x, y)$.

(b) $\exists y \forall x R(x, y)$.

(c) $\forall x \forall y \exists z (R(x, z) \wedge R(y, z))$

Exercise 2: FOL: Is it a model? *(2+3+3 Points)*

Consider the following **first order** formulae

$$\begin{aligned}\varphi_1 &:= \forall x R(x, x)\\ \varphi_2 &:= \forall x \forall y\, R(x, y) \rightarrow (\exists z R(x, z) \wedge R(z, y))\\ \varphi_3 &:= \exists x \exists y\, (\neg R(x, y) \wedge \neg R(y, x))\end{aligned}$$

over signature $\mathcal{S}$ where x, y, z are variable symbols and R is a binary predicate. Give an interpretation

(a) I_1 which is a **model** of $\varphi_1 \wedge \varphi_2$.

(b) I_2 which is **no model** of $\varphi_1 \wedge \varphi_2 \wedge \varphi_3$.

(c) I_3 which is a **model** of $\varphi_1 \wedge \varphi_2 \wedge \varphi_3$.

Exercise 3: FOL: Entailment *(3+3+3 Points)*

Let φ, ψ be first order formulae over signature $\mathcal{S}$. Similar to propositional logic, in predicate logic we write $\varphi \models \psi$ if every model of φ is also a model for ψ. We write $\varphi \equiv \psi$ if both $\varphi \models \psi$ and $\psi \models \varphi$. A *knowledge base* KB is a set of formulae. A model of KB is model for all formulae in KB. We write $KB \models \varphi$ if all models of KB are models of φ. Show or disprove the following entailments.

(a) $(\exists x\, R(x)) \wedge (\exists x\, P(x)) \wedge (\exists x\, T(x)) \models \exists x\, (R(x) \wedge P(x) \wedge T(x))$.

(b) $(\forall x \forall y\, f(x, y) \doteq f(y, x)) \wedge (\forall x\, f(x, \mathbf{c}) \doteq x) \models \forall x\, f(\mathbf{c}, x) \doteq x$.

(c) $(\forall x\, R(x, x)) \;\wedge\; (\forall x \forall y\, R(x, y) \wedge R(y, x) \rightarrow x \doteq y) \;\wedge\; (\forall x \forall y \forall z\, R(x, y) \wedge R(y, z) \rightarrow R(x, z))$
$\models \forall x \forall y\, R(x, y) \vee R(y, x)$.

Hint: Consider order relations. E.g., $a \leq b$ (a less-equal b) and $a|b$ (a divides b).