



# Theory of Distributed Systems

## Exercise Sheet 4

### Exercise 1: Happens Before in Shared Memory

Consider  $n$  processors and  $m$  shared variables. Every processor can access every shared variable with atomic read and write operations (i.e., a process can either read from or write to a shared variable and the system guarantees that such accesses of different processes to the same variable happen atomically). Define a happens before relation similar to the one for message passing.

### Exercise 2: Unique Maximal Cut Preceding a Given Cut

Given a schedule  $S$  with *some* cut  $C$ . Show that there is a unique, maximal consistent cut  $C'$  of  $S$  which precedes the cut  $C$ .

*Remarks: A cut  $C'$  precedes  $C$  if  $C' \subseteq C$ . A cut is maximal with respect to a given property if it contains the most events among all cuts with that property.*

### Exercise 3: Happens Before Relation

Let  $S$  be a schedule with events  $a$ ,  $b$ , and  $c$ . Show that if  $a \not\rightarrow_S b$  and  $a \not\rightarrow_S c$  holds, then there exists some causal shuffle  $S'$  of  $S$  in which  $b$  and  $c$  occur before  $a$ .

### Exercise 4: Logical Clocks

You are given a clique graph on  $n$  nodes. Find two executions  $A$  and  $B$ , in which each node sends exactly one message to every other node, such that

- a) the largest Lamport clock value in  $A$  is as small as possible, and
- b) the largest Lamport clock value in  $B$  is as large as possible.