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Theory of Distributed Systems
Exercise Sheet 8

Exercise 1: Matching

A *matching* of a graph $G = (V, E)$ is a subset of edges $M \subseteq E$ such that no two edges in M are adjacent. A matching is maximal if no edge can be added without violating this property.

Give an algorithm that computes a maximal matching (MM) in $O(\log n)$ rounds w.h.p. in the synchronous message passing model. That is, after the algorithm terminates each node needs to know which of its adjacent edges are part of the maximal matching. *Hint: Try to construct a new graph G', such that solving MIS on G' gives a solution for MM in G.*

Exercise 2: Coloring

Assume we have $C = \alpha(\Delta + 1) \in \mathbb{N}$ colors for some $\alpha \geq 1$. Consider the following algorithm in the synchronous message passing model to color the graph with C colors. Each node v repeats the following steps (corresponding to a phase) until it has a color:

- Let U_v be the set of yet uncolored neighbors of v and let C_v be the set of colors that v's neighbors already chose (initially N_v are all of v's neighbors and $C_v = \emptyset$).
- Node v picks a random number $r_c(v) \in [0, 1]$ for every remaining color $c \in \{1, \ldots, C\} \setminus C_v$ and informs its neighbors about those numbers.
- If $r_c(v) < r_c(u)$ for some $c \in \{1, \ldots, C\} \setminus C_v$ and every $u \in U_v$, then v colors itself with c, informs its neighbors and terminates (if this holds for several c, node v decides on the *'smallest'* of these colors).

(a) What is the probabillity that for some fixed color $c \in \{1, \ldots, C\} \setminus C_v$, $r_c(v) < r_c(u)$ for each uncolored neighbor u of v?

(b) Show that the probability that a node obtains a color in a given phase is at least $1 - e^{-\alpha}$.

(c) Show that the algorithm terminates after $\mathcal{O}\big(1 + \frac{\log n}{\alpha}\big)$ rounds in expectation.

(d) How should we choose the domain of available colors C to get an algorithm that terminates in a constant number of phases in expectation?