University of Freiburg
Dept. of Computer Science
Prof. Dr. F. Kuhn
M. Fuchs, Z. Parsaeian

Theory of Distributed Systems
Exercise Sheet 11

Exercise 1: Sorting Network Short Questions

For each of the following questions, prove or disprove the given claim.

a) The network of width 6 and 12 comparators in Figure 1 below is a sorting network, that is, it sorts each input sequence of numbers correctly.

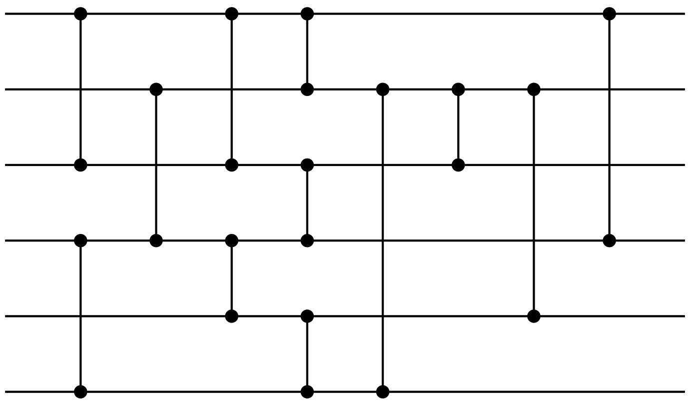

Figure 1: A sorting network of width 6 and 12 comparators.

b) Every correct sorting network needs to have at least one comparator between each two consecutive horizontal lines.

c) A network which contains all $\binom{n}{2}$ comparators between any two of the n horizontal lines, in whatever order they are placed, is a correct sorting network.

d) There is a network which contains $\binom{n}{2}$ comparators that is a correct sorting a network with n wires.

e) Given any correct sorting network, adding another comparator at the end does not destroy the sorting property.

f) Given any correct sorting network, adding another comparator at the front does not destroy the sorting property.

g) Given any correct sorting network, adding another comparator anywhere does not destroy the sorting property.

h) Given any correct sorting network, inverting it (i.e., feeding the input into the output wires and traversing the network "from right to left") results in another correct sorting network.

Exercise 2: Recursive Sorting Network

Suppose that you are given a black-box sorting network of width $n-1$ and that you must adapt it in order to build a sorting network of width n. You are only allowed to add comparators *after* the sorting network (see Figure 2). You can assume that comparators output the maximum value on the bottom wire (i.e., sorting in ascending order starting from the top wire).

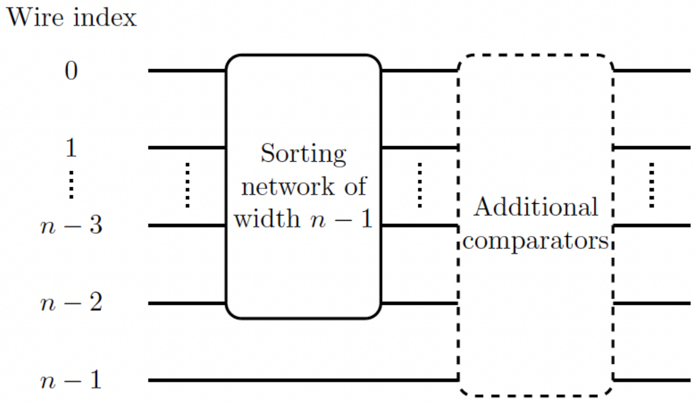

Figure 2: Recursive Sorting Network

a) Find a solution with $n-1$ comparators. Is your solution unique?

b) Show that there is no solution with strictly less than $n-1$ comparators (hence proving that $n-1$ is optimal for building recursive sorting networks in this manner).

c) What is the depth of your construction (ignoring the black-box network)? Show that, whenever n is a power of 2, any such network must have depth $\geq \log_2 n$.

d) Assuming $n = 2^k$, modify your construction (of task a)) to use at most $n-1$ comparators and at most $\log_2 n$ depth.

e) Does your previous constructions also work if you can only add comparators *before* the black-box sorting network?