University of Freiburg
Dept. of Computer Science
Prof. Dr. F. Kuhn
M. Fuchs, A. Mályusz, Z. Parsaeian

Theory of Distributed Systems
Exercise Sheet 1

Exercise 1: Schedules

Consider three nodes, v_1, v_2, and v_3, which are connected via FIFO channels, that is, messages between any two nodes are received in the same order they are sent. For example, if node v_1 sends first message m_1 then m_2 to node v_2, then v_2 will first receive m_1 and then m_2.
Devise **one** possible schedule S which is consistent with the following local restrictions to the three nodes.

- $S|1 = s_{1,3}\ s_{1,3}\ r_{1,2}\ r_{1,3}\ s_{1,2}\ r_{1,2}\ s_{1,3}$,
- $S|2 = s_{2,3}\ s_{2,1}\ r_{2,1}\ s_{2,1}$,
- $S|3 = r_{3,2}\ r_{3,1}\ s_{3,1}\ r_{3,1}\ r_{3,1}$.

$s_{i,j}$ denotes the send event from node i to node j and $r_{j,i}$ denotes the event that node j receives a message from node i.

Exercise 2: The Level Algorithm

Consider the following algorithm between two connected nodes u and v:

The two nodes maintain levels ℓ_u and ℓ_v, which are both initialized to 0. One round of the algorithm works as follows:

1. Both nodes send their current level to each other
2. If u receives level ℓ_v from v, u updates its level to $\ell_u := \max\{\ell_u, \ell_v + 1\}$. If the message to node u is lost, node u does not change its level ℓ_u. Node v updates its level ℓ_v in the same (symmetric) way.

If the level algorithm runs for r rounds:

(a) What can you say about the level of the two nodes?

(b) If all messages succeed, what can you say about the level of the two nodes?

(c) In what case the level of a node is at least one?

Exercise 3: The Randomized Two Generals Algorithm

Now assume we have two nodes u and v running the following modified protocol:

- Each node has an input $x_u, x_v \in \{0, 1\}$.
- Node u picks a random number $R \in \{1, \ldots, r\}$ uniformly at random.

- The nodes run the **Level Algorithm** (as in Exercise 2) for r rounds (In each message, both nodes also include their inputs and node u also includes the value of R.).
- At the end, a node outputs 1 if:
  - It knows that both inputs are 1,
  - It knows the value of R,
  - Its own level is at least R.
- Otherwise, it outputs 0.

If the level algorithm runs for r rounds:

(a) If at least one input is 0, what is the output of the two nodes?

(b) If both inputs are 1:

- what is the output if no message is lost?
- under what circumstances the output of the two nodes is not the same value?

(c) If both inputs are 1, what is the probability that both nodes output the same value?

(d) Using the same technique as in the impossibility proof for the deterministic Two Generals Problem (discussed in the lecture), prove a lower bound for the error probability.