



Theoretical Computer Science - Bridging Course

Exercise Sheet 4

Due: Tuesday, 20th of May 2025, 12:00 pm

Exercise 1: CFGs

(10 Points)

Give a context free grammar for each of the following languages.

- $L_1 = \{a^i b^j \mid 0 < i \leq j\}$
- $L_2 = \{a^{2^n} b^n \mid n > 0\}$
- $L_3 = \{a^* w c^k \mid w \in \{a, b\}^*, \text{ and } k \text{ is the number of } a\text{'s in } w\}$
- $L_4 = \{a^k b^{3k} \mid k \geq 0\}$
- $L_1 \circ L_2$
- $L_1 \cup L_2$

Exercise 2: PDAs

(6 Points)

Construct a PDA for the following languages

- L_2 in exercise 1.
- L_3 in exercise 1.
- $L_5 = \{a^n b^{2m} b a^n \mid m, n > 0\}$ over the alphabet $\Sigma = \{a, b\}$.

Exercise 3: Chomsky Normal Form

(4 Points)

Given the following CFG:

$$S \rightarrow ASA \mid A \mid \epsilon$$

$$A \rightarrow 00 \mid \epsilon$$

1. What is the language the above CFG recognizes?
2. Convert the following CFG into an equivalent CFG in Chomsky normal form, by following the procedure given in the lecture.

Exercise 4: Closure in CFL

(Bonus Points)

1. Show that the context-free languages are closed under union, concatenation and Kleene star.
Hint: try to prove that the context-free languages are closed under the above operators via creating the appropriate grammars.
2. Knowing that $L_6 = \{a^i b^j c^k \mid i < j\}$ is a context free language, are context free languages closed under intersection?
Hint: Use the fact that $L_7 = \{a^i b^j c^k \mid i < j \text{ and } i < k\}$ is not a context free language.