



Theoretical Computer Science - Bridging Course

Exercise Sheet 8

Due: Tuesday, 24th of June 2025, 12:00 pm

Exercise 1: Class \mathcal{P}

(3+3+4 Points)

\mathcal{P} is the set of languages (\cong decision problems) which can be decided by an algorithm whose runtime can be bounded by $p(n)$, where p is a polynomial and n the size of the respective input (problem instance). Show that the following languages are in the class \mathcal{P} . Since it is typically easy (i.e. feasible in polynomial time) to decide whether an input is well-formed, your algorithm only needs to consider well-formed inputs. Use the \mathcal{O} -notation to bound the run-time of your algorithm.

- (a) $\text{PALINDROME} := \{w \in \{0,1\}^* \mid w \text{ is a Palindrome}\}$
- (b) $4\text{-CLIQUE} := \{\langle G \rangle \mid G \text{ has a clique of size at least 4}\}$
- (c) $5\text{-VERTEXCOVER} := \{\langle G \rangle \mid G \text{ has a vertex cover of size at most 5}\}.$

Remarks:

- In both problems G is an undirected, simple graph.
- A *clique* in a graph $G = (V, E)$ is a set $C \subseteq V$ such that for all $u, v \in C : \{u, v\} \in E$.
- A *vertex cover* of $G = (V, E)$ is a subset $C \subseteq V$ of nodes, such that for all $\{u, v\} \in E$ it holds that $u \in C$ or $v \in C$.

Exercise 2: The Class \mathcal{NP}

(5+5 Points)

Show that the following problems (languages) are in class \mathcal{NP} .

- (a) Given a graph $G = (V, E)$ and an integer k , it is required to determine whether G contains a clique of size at least k , hence consider the following problem:
 $\text{CLIQUE} := \{\langle G, k \rangle \mid G \text{ has a clique of size at least } k\}.$

- (b) A *hitting set* $H \subseteq \mathcal{U}$ for a given universe \mathcal{U} and a set $S = \{S_1, S_2, \dots, S_m\}$ of subsets $S_i \subseteq \mathcal{U}$, fulfills the property $H \cap S_i \neq \emptyset$ for $1 \leq i \leq m$ (H 'hits' at least one element of every S_i).

Given a universe set \mathcal{U} , a set S of subsets of \mathcal{U} , and a positive integer k , it is required to determine whether \mathcal{U} contains a hitting set of size at most k , hence consider the following problem:

$\text{HITTINGSET} := \{\langle \mathcal{U}, S, k \rangle \mid \text{universe } \mathcal{U} \text{ has subset of size } \leq k \text{ that hits all sets in } S \subseteq 2^{\mathcal{U}}\}.$ ¹

¹The power set $2^{\mathcal{U}}$ of some ground set \mathcal{U} is the set of all subsets of \mathcal{U} . So $S \subseteq 2^{\mathcal{U}}$ is a collection of subsets of \mathcal{U} .