



Theoretical Computer Science - Bridging Course

Exercise Sheet 11

Due: Tuesday, 15th of July 2025, 12:00 pm

Exercise 1: Construct Formulae

(1+1+1 Points)

Let $\mathcal{S} = \langle \{x, y, z\}, \emptyset, \emptyset, \{R\} \rangle$ be a signature. Translate the following sentences of first order formula over \mathcal{S} into idiomatic English i.e. how would that native speaker read the formulas?

Use $R(x, y)$ as statement 'x is a part of y'.

- (a) $\exists x \forall y R(x, y)$
- (b) $\exists y \forall x R(x, y)$
- (c) $\forall x \forall y \exists z (R(x, z) \wedge R(y, z))$

Exercise 2: FOL: Is it a model?

(2+3+3 Points)

Consider the following **first order** formulae

$$\begin{aligned}\varphi_1 &:= \forall x R(x, x) \\ \varphi_2 &:= \forall x \forall y R(x, y) \rightarrow (\exists z R(x, z) \wedge R(z, y)) \\ \varphi_3 &:= \exists x \exists y (\neg R(x, y) \wedge \neg R(y, x))\end{aligned}$$

over signature \mathcal{S} where x, y, z are variable symbols and R is a binary predicate. Give an interpretation

- (a) I_1 which is a **model** of $\varphi_1 \wedge \varphi_2$.
- (b) I_2 which is **no model** of $\varphi_1 \wedge \varphi_2 \wedge \varphi_3$.
- (c) I_3 which is a **model** of $\varphi_1 \wedge \varphi_2 \wedge \varphi_3$.

Exercise 3: FOL: Entailment

(3+3+3 Points)

Let φ, ψ be first order formulae over signature \mathcal{S} . Similar to propositional logic, in predicate logic we write $\varphi \models \psi$ if every model of φ is also a model for ψ . We write $\varphi \equiv \psi$ if both $\varphi \models \psi$ and $\psi \models \varphi$. A *knowledge base* KB is a set of formulae. A model of KB is model for all formulae in KB . We write $KB \models \varphi$ if all models of KB are models of φ . Show or disprove the following entailments.

- (a) $(\exists x R(x)) \wedge (\exists x P(x)) \wedge (\exists x T(x)) \models \exists x (R(x) \wedge P(x) \wedge T(x))$.
- (b) $(\forall x \forall y f(x, y) \doteq f(y, x)) \wedge (\forall x f(x, \mathbf{c}) \doteq x) \models \forall x f(\mathbf{c}, x) \doteq x$.
- (c) $(\forall x R(x, x)) \wedge (\forall x \forall y R(x, y) \wedge R(y, x) \rightarrow x \doteq y) \wedge (\forall x \forall y \forall z R(x, y) \wedge R(y, z) \rightarrow R(x, z)) \models \forall x \forall y R(x, y) \vee R(y, x)$.

Hint: Consider order relations. E.g., $a \leq b$ (a less-equal b) and $a|b$ (a divides b).