University of Freiburg
Dept. of Computer Science
Prof. Dr. F. Kuhn

Theoretical Computer Science

19th of February 2025, 10:00 - 12:00

Name: ....

Matriculation No.: ....

Signature: ....

Do not open or turn until told so by the supervisor!

- Write your **name** and **matriculation number** on this page and **sign** the document.
- Your **signature** confirms that you have answered all exam questions yourself without any help, and that you have notified exam supervision of any interference.
- You are allowed to use a summary of **4 handwritten A4 pages**.
- **No electronic devices** are allowed.
- Write legibly and only use a pen (ink or ball point). Do **not use red**! Do **not use a pencil**!
- You may write your answers in **English**.
- Only **one solution per task** is considered! Make sure to strike out alternative solutions, otherwise the one yielding the minimal number of points is considered.
- **Detailed steps** might help you to get more points in case your final result is incorrect.
- The keywords **Show...**, **Prove...**, **Explain...** or **Argue...** indicate that you need to prove or explain your answer carefully and in sufficient detail.
- The keywords **Give...**, **State...** or **Describe...** indicate that you need to provide an answer solving the task at hand but without proof or deep explanation (except when stated otherwise).
- You may use information given in a **Hint** without further explanation.
- **Read each task thoroughly** and make sure you understand what is expected from you.
- **Raise your hand** if you have a question regarding the formulation of a task or if you need additional sheets of paper.
- A total of **60 points** is sufficient to pass and a total of **120 points** is sufficient for the best grade.
- Write your name on **all sheets**!

Task	1	2	3	4	5	**Total**
Maximum	28	22	10	41	19	120
Points						

1: Regular Languages *(28 Points)*

1. Design a DFA for the language L over $\Sigma = \{a, b\}$ such that L consists of all strings where the number of occurrences of b is divisible by 4. *(8 Points)*

2. Construct a regular expression for the language L over $\Sigma = \{a, b, c\}$ where every string contains at most one occurrence of c and ends with a. *(10 Points)*

3. Show that the language $L = \{a^k b^n \mid k, n \geq 0 \text{ and } n = c \cdot k\}$, for a given constant integer $c \geq 1$, is not regular by using the Pumping Lemma for regular languages. *(10 Points)*

2: Context-Free Languages *(22 Points)*

1. Give a context-free grammar for the language $L = \{a^n b^m a^k \mid m, n, k \geq 0 \text{ and } n \leq k\}$. Describe why your grammar correctly generates this language. *(10 Points)*

2. Construct a Pushdown Automaton (PDA) for the language $L = \{a^n b^m c^m d^n \mid n, m \geq 1\}$. You do not need to give an exact formal description of the PDA, but you need to give sufficient details so that the individual step, states, and state transitions become clear. *(12 Points)*

3: Turing Machines *(10 Points)*

1. Give a Turing Machine that takes a binary number as input and computes its NOT. Your Turing Machine description does not need to use the exact formalism from the lecture, it however has to contain all the details that a formal description would entail. *(10 Points)*

4: Complexity Theory *(41 Points)*

1. A vertext cover of a graph $G = (V, E)$ is a set of nodes $S \subseteq V$ so that for every edge $\{u, v\} \in E$, at least one of the two endpoints is in S (i.e., $\{u, v\} \cap S$). Show that the k-Vertex Cover Problem (i.e., the problem of determining if a given graph has a vertex cover of size at most k) is in *NP*. *(12 Points)*

2. Compare the asymptotic growth rates of the following functions: $f(n) = n^3$, $g(n) = 2^n$, and $h(n) = n \log n$. Rank them in order of increasing growth rate and describe your ranking. *(10 Points)*

3. (a) Describe what P and NP are and what the relationship between them is. *(7 Points)*

   (b) Consider the problem of 3-Clique (deciding whether a graph has a clique of size at least 3). Is this problem in P? Is it in NP? Explain your answer. *(12 Points)*

5: Propositional Logic *(19 Points)*

1. Convert the formula $(\neg p \lor q) \lor (r \land \neg q)$ into Conjunctive Normal Form (CNF) using logical equivalences. *(6 Points)*

2. Construct a truth table for the formula: $(p \lor q) \to (\neg p \lor r)$. *(4 Points)*

3. Consider the following propositional logic formulas: *(9 Points)*

   - $\phi_1 = (p \lor q) \land (\neg p \lor r)$
   - $\phi_2 = (p \to q) \lor (q \to p)$
   - $\phi_3 = (p \lor \neg p) \land (q \lor \neg q)$

   For each formula, determine whether it is:

   - Always true (a **tautology**)
   - Satisfiable but not always true (**satisfiable but not a tautology**)
   - Never true (**unsatisfiable**)