# Warming up

for TCS Bridging Course

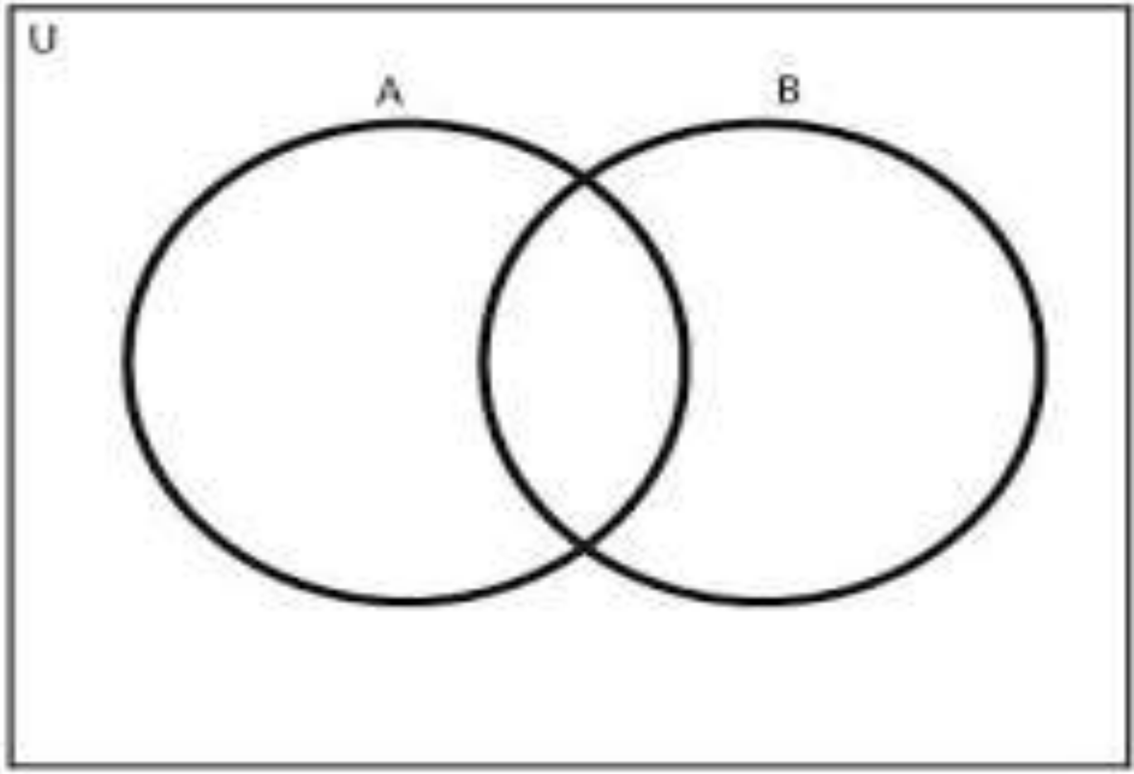
- ***Mathematical objects, tools, notions:***
- Sets
- Sequences
- Functions
- Graphs
- Strings and languages

- ***Types of Proof:***
- By construction
- By contradiction
- By induction
- By counterexample

- The alphabet set $\sum$ ={ a, c, n, o, r}
- A={ no, corona}
- B= {no, corona, roar, ac}
- Is A $\subseteq$ B?
- Is B $\subseteq$ A?
- A $\cup$ $B$?
- A $\cap$ B?
- B \ A?
- A $\backslash$B?

- For any two sets A and B,

$A \Delta B = \emptyset \Leftrightarrow A = B$




*Proof*

$\Rightarrow)$: $A \Delta B = (A \setminus B) \cup (B \setminus A) = \emptyset$

$(A \setminus B) = \emptyset$ and $(B \setminus A) = \emptyset$

$A \subseteq B$ $\quad$ $B \subseteq A$

$A = B$

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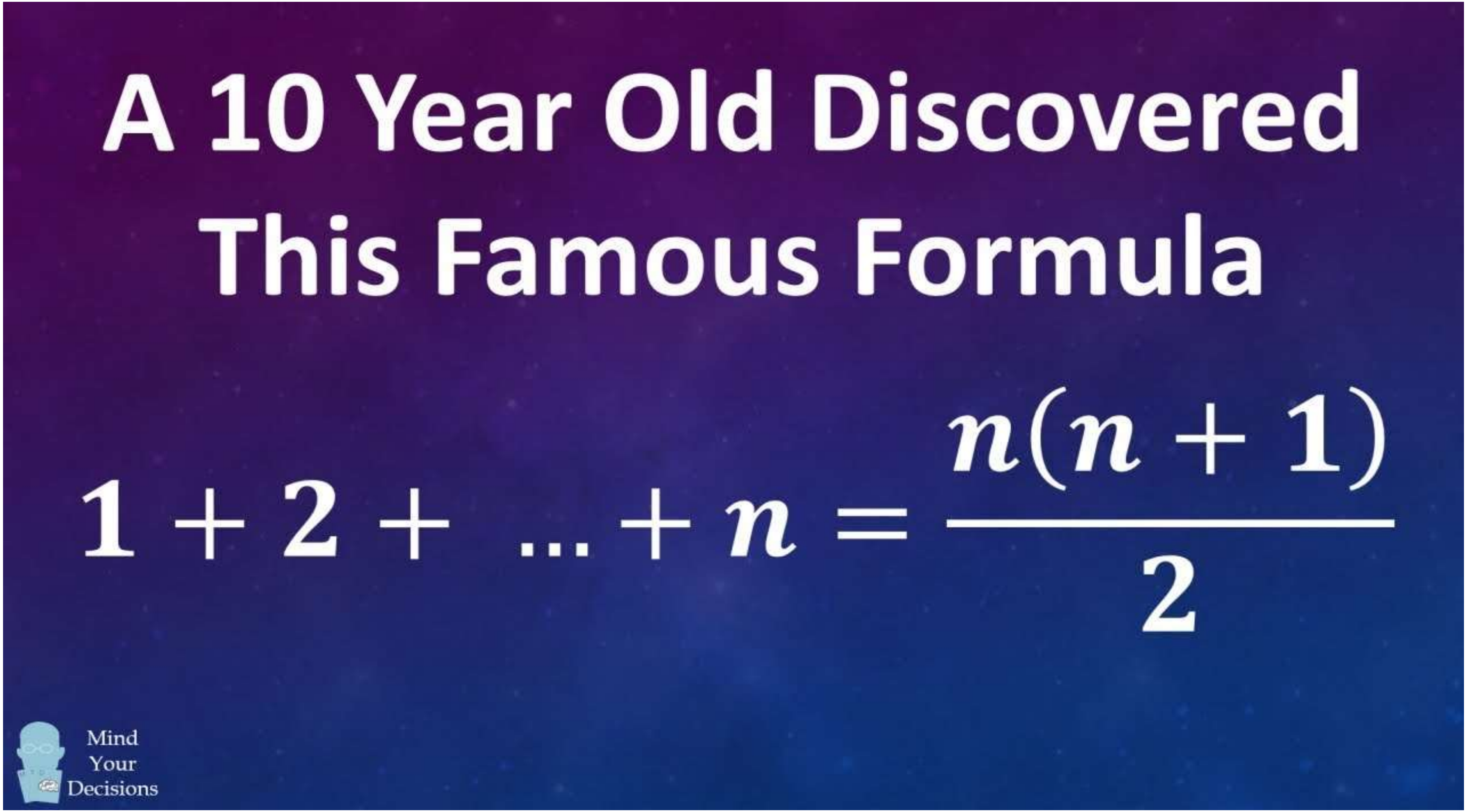
A 10 Year Old Discovered This Famous Formula
$$1 + 2 + \ldots + n = \frac{n(n+1)}{2}$$
Mind
Your
Decisions

# Induction

- **Goal:** for an integer $n \geq 0$, use mathematical induction to prove a statement holds true for all values of *n.*

<u>2 STEPS:</u>

- **Base case** :

prove the statement true for $n = 0$

- **Induction step**:

assume the statement holds for any given case $n= k$, where $k \geq 0$ and use this assumption to prove the statement true for $n = k + 1$.

- Use proof by induction to prove
- *1+2+…+n=* $\frac{n(n+1)}{2}$, for n $\geq$ 1
- Base step: for n=1, we have $\frac{1(1+1)}{2}$ = 1
- Inductive hypothesis: assume for any case *n=k* holds , where k is some integer k $\geq$1

i.e. *1+2+…+k* = $\frac{k(k+1)}{2}$, where k is some integer k $\geq$1

Now, let's prove the statement true for *n=k+1*

*i.e. 1+2+…+(k+1)=* $\frac{(k+1)(k+2)}{2}$ (is it true?)

*1+2+…k + (k+1)=* $\frac{k(k+1)}{2}$ + *(k+1 )=* $\frac{k(k+1)+2(k+1)}{2}$ = $\frac{(k+1)(k+2)}{2}$ (Yes!)

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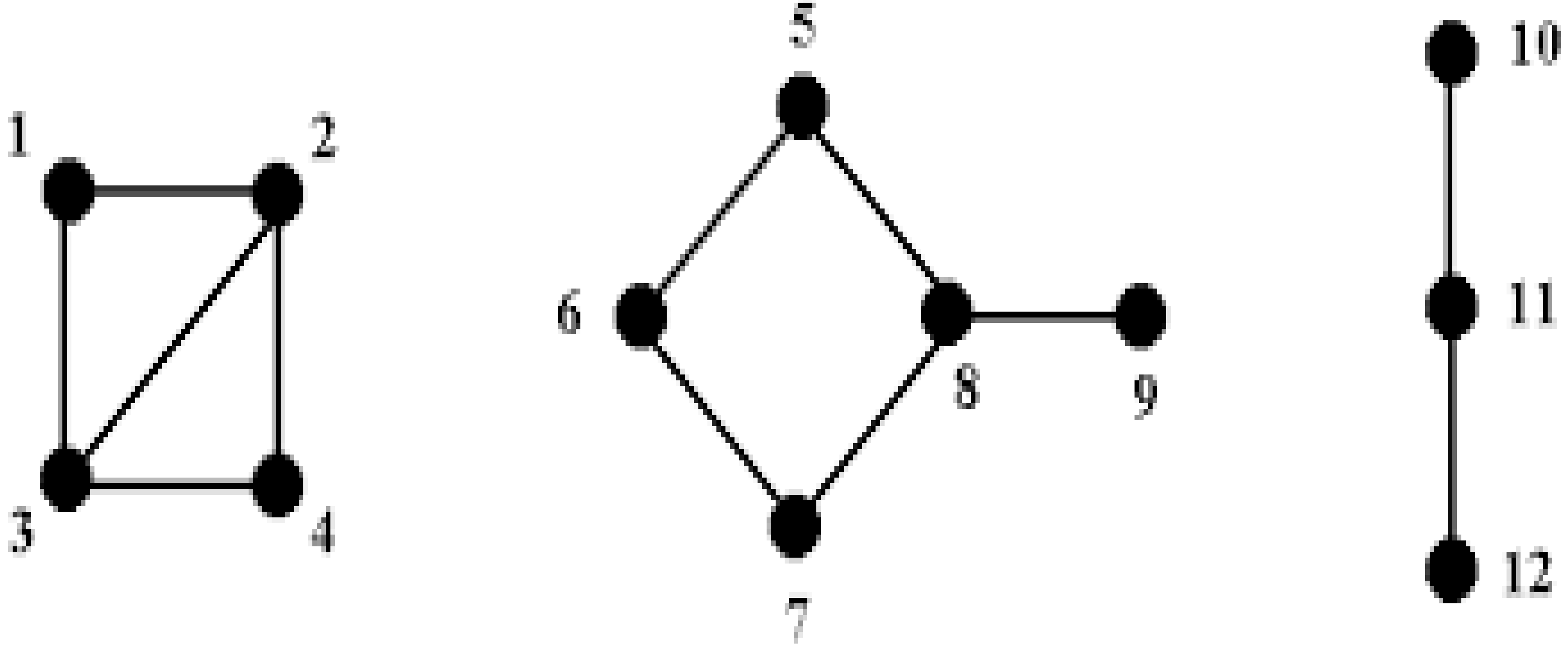
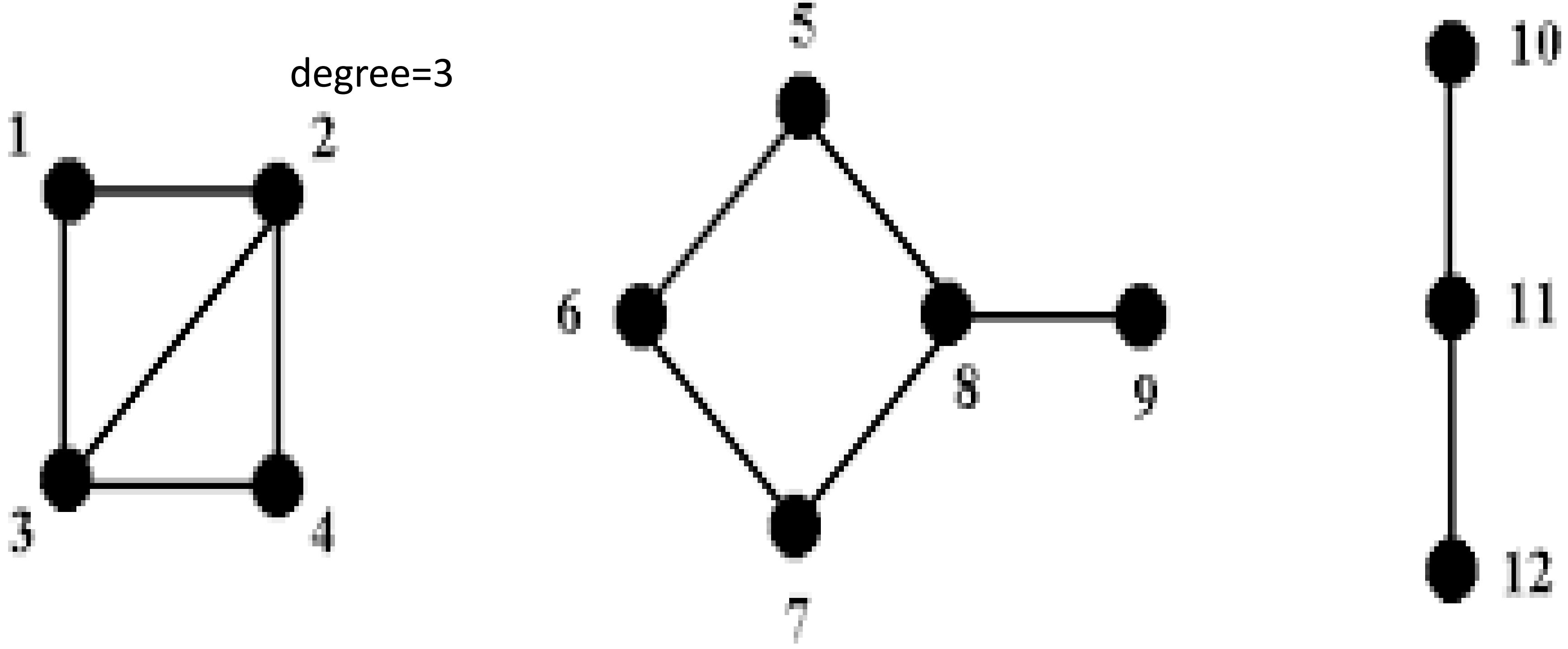
# Graphs

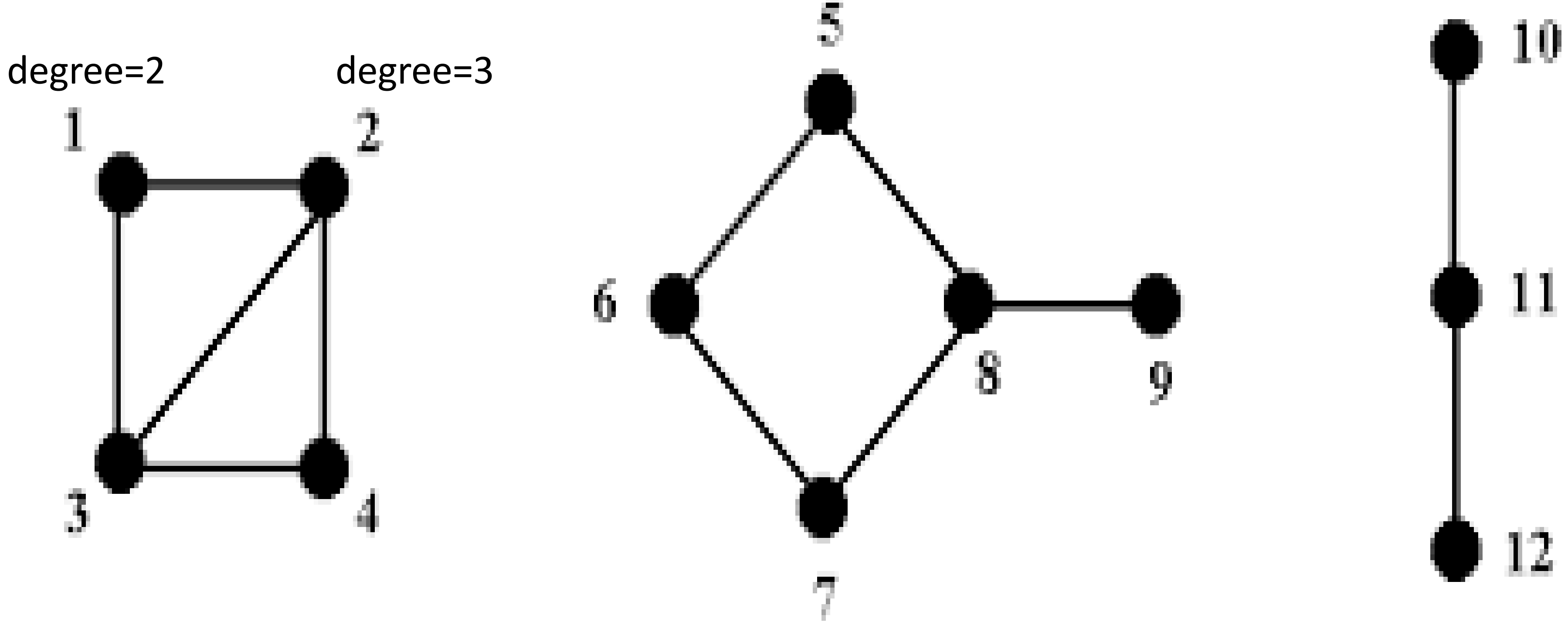
We write G=(V,E).



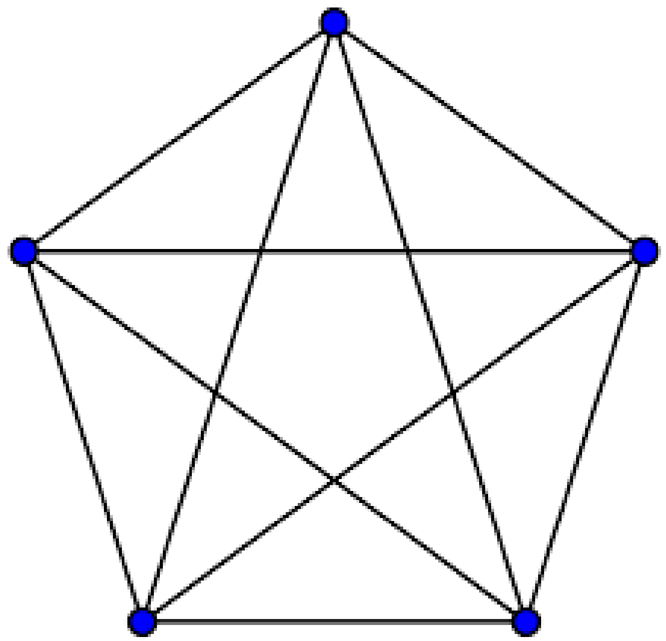
# Graphs



# Graphs



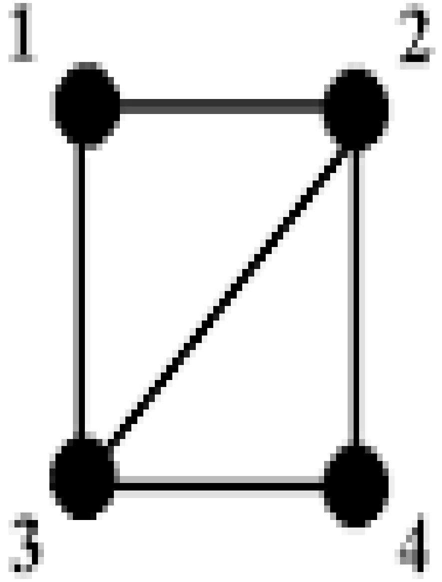
How many edges in a *complete graph on **n** vertices*?



n(n-1)/2 edges!

Don't count each edge twice

- How many edges are there in a simple graph G= (V, E) ?
- $\sum_{v \in V} degree\ (v) = 2\ |E|$ (Handshaking Lemma)
- Each edge contributes 2 to the sum on the left.



# Can you?

- Draw a graph on 5 nodes such that each node is of degree 3.

# Can you?

Draw a graph on 5 nodes such that each node is of degree 3

- <u>Solution:</u>  you can't!

- Sum of all degrees= 5 x 3= 15

- See you Next Week !