Albert-Ludwigs-Universität, Inst. für Informatik
Prof. Dr. Fabian Kuhn
H. Ghodselahi, Y. Maus
December 10, 2014

Algorithm Theory, Winter Term 2014/15
Problem Set 8

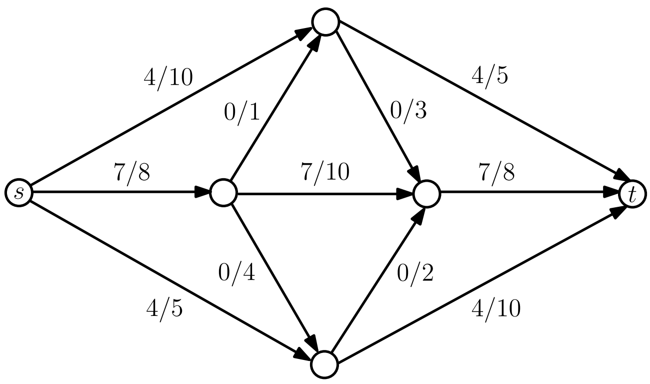
hand in (hard copied) by Thursday, 10:00, December 18, 2014, either before the lecture or in the box corresponding to your group in building no. 51.

Exercise 1: Binomial Heap (3 points)

Consider a binomial heap data structure. Give a potential function which shows that the amortized cost of *insert* is $O(\log n)$ and the amortized time of *delete-min* is $O(1)$.

Exercise 2: Ford-Fulkerson (3.5+1+2+2.5 points)

Consider the following flow network, where for each edge, the capacity (second larger number) and a current flow value (first smaller number) are given.



a) Find a maximal flow in the given network with the help of the Ford Fulkerson algorithm.

   Draw the residual graph with all the residual capacities in all steps.

b) Give a minimal $s-t$ cut.

c) Is the following statement true or false?

   If all edges in a flow network have distinct capacities, then there is a unique maximum flow.

   Justify your answer with a (short) proof or give a counterexample.

d) You are given a (connected) directed graph $G = (V, E)$, with positive integer capacities on each edge, a designated source $s \in V$, and a designated sink $t \in V$. Additionally you are given a current maximum $s-t$ flow $f : E \to \mathbb{R}_{\geq 0}$.

   Now suppose we increase the capacity of one specific edge $e_0 \in E$ by one unit. Show how to find a maximum flow in the resulting capacitated graph in time $\mathcal{O}(|E|)$.