



Chapter 8

Online Algorithms

Algorithm Theory
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Online Computations

- Sometimes, an algorithm has to start processing the input before the complete input is known
- For example, when storing data in a data structure, the sequence of operations on the data structure is not known

Online Algorithm: An algorithm that has to produce the output step-by-step when new parts of the input become available.

Offline Algorithm: An algorithm that has access to the whole input before computing the output.

- Some problems are inherently online
 - Especially when real-time requests have to be processed over a significant period of time

Competitive Ratio

- Let's again consider optimization problems
 - For simplicity, assume, we have a minimization problem

Optimal offline solution OPT(I):

- Best objective value that an offline algorithm can achieve for a given input sequence I

Online solution ALG(I):

- Objective value achieved by an online algorithm ALG on I

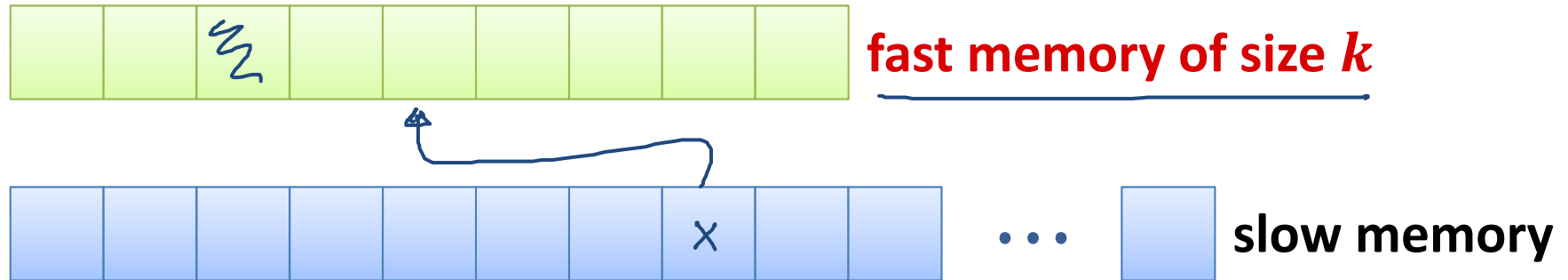
Competitive Ratio: An algorithm has **competitive ratio $c \geq 1$** if

$$\mathbf{ALG(I) \leq c \cdot OPT(I) + \alpha.}$$

- If $\alpha \leq 0$, we say that ALG is strictly c -competitive.

Paging Algorithm

Assume a simple memory hierarchy:



If a memory page has to be accessed:

- Page in fast memory (hit): take page from there
- Page not fast memory (miss): leads to a page fault
- Page fault: the page is loaded into the fast memory and some page has to be evicted from the fast memory
- Paging algorithm: decides which page to evict
- Classical online problem: we don't know the future accesses

Paging Strategies

Least Recently Used (LRU):

- Replace the page that hasn't been used for the longest time

First In First Out (FIFO):

- Replace the page that has been in the fast memory longest

Last In First Out (LIFO):

- Replace the page most recently moved to fast memory

Least Frequently Used (LFU):

- Replace the page that has been used the least

Longest Forward Distance (LFD):

optimal offline strategy

- Replace the page whose next request is latest (in the future)
- LFD is **not an online strategy!**