Albert-Ludwigs-Universität, Inst. für Informatik
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Theoretical Computer Science - Bridging Course
Winter Term 2016
Exercise Sheet 1

hand in (electronically or hard copy) by 23:59, Sunday, October 30th, 2016

Exercise 1: Proof by Induction (5 points)

Let $x \geq -1$. Show that $(1+x)^n \geq 1+nx$ for all $n \in \mathbb{N}$ using induction on n.

Exercise 2: Partition of a Set (5 points)

A partition of a set A is a collection of sets $B_i, i \in \{1, \ldots, n\}$ such that

$$B_1 \cup \ldots \cup B_n = A \text{ and } B_i \cap B_j = \emptyset \text{ for } i \neq j.$$

Show that $B_i := \{3k + i \,|\, k \in \mathbb{Z}\}$, $i \in \{1, 2, 3\}$ is a partition of $\mathbb{Z}$.

Hint: $\mathbb{Z}$ is the set of integers. In order to proof that two sets are equal consider an arbitrary element from one set and show that it is contained in the other set and vice versa.

Exercise 3: Counting Edges in Acyclic Graph (5 points)

A tree is an acyclic, connected, simple graph. Show that a tree with $n \geq 1$ nodes has $n-1$ edges. A forest is a graph consisting of several unconnected trees. Show that a forest consisting of k components has $n-k$ edges.

Hint: A simple graph is an unweighted, undirected graph containing no self-loops or multiple edges.

Exercise 4: Nodes with Identical Degrees (5 points)

Show that every simple graph with two or more nodes contains two nodes with the same degree.