Albert-Ludwigs-Universität, Inst. für Informatik
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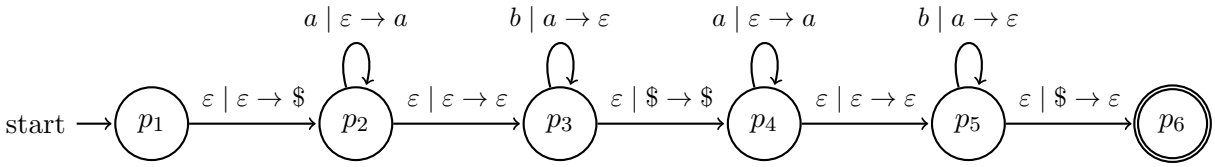
Theoretical Computer Science - Bridging Course
Winter Term 2016
Exercise Sheet 5

Hand in (electronically or hard copy) before your weekly meeting but not later than 23:59, Wednesday, November 30, 2016

Exercise 1: The Push Down Automaton (3+4 points)

(a) Let $\Sigma = \{a, b\}$, use set notation to describe the language that is recognized by the following PDA.



(b) Let $\Sigma = \{0, 1\}$ and let L be the collection of strings that contain at least one 1 in their second half. That is, $L = \{uv \mid u \in \Sigma^*, v \in \Sigma^* 1 \Sigma^*, |u| \geq |v|\}$. Give a PDA that recognizes L.

Exercise 2: Context-Free Languages and Set Operations (3+3 points)

(a) Show that context-free languages are not closed under taking intersections (i.e., the intersection of two context-free languages is not necessarily context free).
Hint: You can use that the language $\{a^i b^i c^i | i \geq 0\}$ is not context-free.

(b) Show that context-free languages are not closed under taking complements.
Hint: You can use DeMorgan's law and the fact that the set of context-free languages is closed under performing union operations.

Exercise 3: Pumping Lemma for Context-Free Languages (3+4 points)

(a) Let $L = \{w \in \Sigma^* \mid$ number of 1s equals number of 2s, and number of 3s equals number of 4s in $w\}$. Here, $\Sigma = \{1, 2, 3, 4\}$. Show that L is not context-free.

(b) Let $L = \{wtw^R \mid w, t \in \{0, 1\}^*, |w| = |t|\}$. Prove that L is not a context-free language. Here, w^R denotes the reverse on a string w, e.g., if $w = 101110$, then $w^R = 011101$.