Albert-Ludwigs-Universität, Inst. für Informatik
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Theoretical Computer Science - Bridging Course
Winter Term 2017
Exercise Sheet 1

hand in (electronically or hard copy) by 12:15 pm, Monday, October 30th, 2017

Exercise 1: Induction *(6 Points)*

Prove the following summation formula by induction on n:

For all natural numbers $n \geq 1$ it holds: $\sum\limits_{k=1}^{n} k^2 = \frac{1}{6}n(n+1)(2n+1)$

Exercise 2: Sets & More *(8 Points)*

Either proof the following statements or show that they are not true.

1. $\{(x,y) \in \mathbb{Z} \times \mathbb{Z} \mid x+y=0\} \subseteq \{(x,y) \in \mathbb{Z} \times \mathbb{Z} \mid x \cdot y = 0\}$
2. $A \setminus (B \cap C) = (A \setminus B) \cup (A \setminus C)$
3. $A \subsetneq B$ implies $C \cup A \subsetneq C \cup B$.

Exercise 3: Where is the even degree node? *(6 Points)*

A *simple graph* is a graph without self loops, i.e., every edge of the graph is an edge between to distinct nodes. The degree $d(v)$ of a node $v \in V$ in an undirected graph $G = (V, E)$ is the number of its neighbors, i.e,

$$d(v) = |\{u \in V \mid \{v,u\} \in E\}|.$$

Show that every simple graph with an odd number of nodes contains a node with even degree.

Hint: Consider the sum $D = \sum_{v \in V} d(v)$ of all degrees. Is D odd or even?