Albert-Ludwigs-Universität, Inst. für Informatik
Prof. Dr. Fabian Kuhn
P. Bamberger, Y. Maus,

Theoretical Computer Science - Bridging Course
Summer Term 2017
Exercise Sheet 6

Hand in (electronically or hard copy) by 12:15 pm, December 4th, 2017

Exercise 1: Designing a Turing Machine *(6 Points)*

Design a Turing machine which accepts the language $L = \{w\#\overline{rev}(w) \mid w \in \{0,1\}^*\}$ where $\overline{rev}$ denotes the reverse complement, i.e., $\overline{rev}(a_1a_2 \dots a_{n-1}a_n) = \overline{a_n}\ \overline{a_{n-1}} \dots \overline{a_2}\ \overline{a_1}$ with $\overline{0} = 1$ and $\overline{1} = 0$.

Remark: It is sufficient to give a detailed description of the Turing Machine. You do not need to give a formal definition.

Exersive 2: Semi-Decidable vs. Recursively Enumerable *(5 Points)*

Very often people in computer science use the terms *semi-decidable* and *recursively enumerable* equivalently. The following exercise shows in which way they actually are equivalent. We first recall the definition of both terms.

A language L is *semi-decidable* if there is a Turing machine which accepts every $w \in L$ and does not accept any $w \notin L$ (this means the TM can either reject $w \notin L$ or simply not stop for $w \notin L$).

A language is *recursively enumerable* if there is a Turing machine which eventually outputs every word $w \in L$ and never outputs a word $w \notin L$.

(a) Show that any recursively enumerable language is semi-decidable.

(b) Show that any semi-decidable language is recursively enumberable.

Exercise 3: Halting Problem *(3+2+2+2 Points)*

The *special halting problem* is defined as

$$H_s = \{\langle M \rangle \mid \langle M \rangle \text{ encodes a TM and } M \text{ halts on } \langle M \rangle\}.$$

(a) Show that H_s is undecidable.

Hint: Assume that M is a TM which decides H_s and then construct a TM which halts iff M does not halt. Use this construction to find a contradiction.

(b) Show that the special halting problem is recursively enumerable.

(c) Show that the complement of the special halting problem is not recursively enumerable.

Hint: What can you say about a language L if L and its complement are recursively enumerable? (if you make some observation for this, also prove it)

(d) Let L_1 and L_2 be recursively enumerable languages. Is $L_1 \setminus L_2$ recursively enumerable as well?

(e) Is $L = \{w \in H_s \mid |w| \leq 1742\}$ decidable? Explain your answer!