Albert-Ludwigs-Universität, Inst. für Informatik
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Theoretical Computer Science - Bridging Course
Summer Term 2017
Exercise Sheet 7

Hand in (electronically or hard copy) by 12:15 pm, December 11th, 2017

Exercise 1: Decidability *(7 Points)*

Consider the following language

$$A_{\text{all}} = \{\langle M \rangle \mid \text{TM } M \text{ accepts all inputs}\}$$

Show that A_{all} is not decidable.
Hint: Assume that a TM M_c decides A_{all}. Then run the Turing machine M_c on some very wisely chosen input to decide the halting problem.

Exercise 2: Landau Notation *(2+2+3 Points)*

Prove or disprove the following statements

(a) $100\sqrt{n} \in O(0.001 \cdot n)$.

(b) $\log_2 3^n \in O(n)$.

(c) $2 \cdot n \in O(10 \cdot \sqrt{n})$.

Remark: There are thousands of similar exercises on the 'net'. Go through some of them to practice for the exam.

Exercise 3: Sort Functions by Asymptotic Growth *(6 Points)*

Sort the following functions by asymptotic growth using the $\mathcal{O}$-notation. Write $g <_{\mathcal{O}} f$ if $g \in \mathcal{O}(f)$ *and* $g \notin \mathcal{O}(f)$. Write $g =_{\mathcal{O}} f$ if $f \in \mathcal{O}(g)$ *and* $g \in \mathcal{O}(f)$.

n^2	$\sqrt{n}$	2^n	$\log(n^2)$
3^n	n^{100}	$\log(\sqrt{n})$	$(\log n)^2$
$\log n$	$10^{100}n$	$n!$	$n \log n$
$n \cdot 2^n$	n^n	$\sqrt{\log n}$	n