Albert-Ludwigs-Universität, Inst. für Informatik
Prof. Dr. Fabian Kuhn
Philipp Bamberger, Yannic Maus

Theoretical Computer Science - Bridging Course
Summer Term 2018
Exercise Sheet 6

for getting feedback submit (electronically) before the start of the tutorial on 3rd of December 2018.

Exercise 1: Semi-Decidable vs. Recursively Enumerable *(5 Points)*

Very often people in computer science use the terms *semi-decidable* and *recursively enumerable* equivalently. The following exercise shows in which way they actually are equivalent. We first recall the definition of both terms.

A language L is *semi-decidable* if there is a Turing machine which accepts every $w \in L$ and does not accept any $w \notin L$ (this means the TM can either reject $w \notin L$ or simply not stop for $w \notin L$).

A language is *recursively enumerable* if there is a Turing machine which eventually outputs every word $w \in L$ and never outputs a word $w \notin L$.

(a) Show that any recursively enumerable language is semi-decidable.

(b) Show that any semi-decidable language is recursively enumberable.

Exercise 2: Halting Problem *(3+2+2+2 Points)*

The *special halting problem* is defined as

$$H_s = \{\langle M\rangle \mid \langle M\rangle \text{ encodes a TM and } M \text{ halts on } \langle M\rangle\}.$$

(a) Show that H_s is undecidable.

Hint: Assume that M is a TM which decides H_s and then construct a TM which halts iff M does reject. Use this construction to find a contradiction.

(b) Show that the special halting problem is recursively enumerable.

(c) Show that the complement of the special halting problem is not recursively enumerable.

Hint: What can you say about a language L if L and its complement are recursively enumerable? (if you make some observation for this, also prove it)

(d) Let L_1 and L_2 be recursively enumerable languages. Is $L_1 \setminus L_2$ recursively enumerable as well?

(e) Is $L = \{w \in H_s \mid |w| \leq 1742\}$ decidable? Explain your answer!

Exercise 3: Undecidability *(5 Points)*

Fix an enumeration of all Turing machines (that have input alphabet Σ): $\langle M_1\rangle, \langle M_2\rangle, \langle M_3\rangle, \dots$.
Fix also an enumeration of all words over Σ: $w_1, w_2, w_3, \dots$.
Prove that language $L = \{w \in \Sigma^* \mid w = w_i, \text{ for some } i, \text{ and } M_i \text{ does not accept } w_i\}$ is not Turing recognizable.
Hint: Try to find a contradiction to the existence of a Turing machine that recognizes L.