Albert-Ludwigs-Universität, Inst. für Informatik
Prof. Dr. Fabian Kuhn
Philipp Bamberger, Yannic Maus

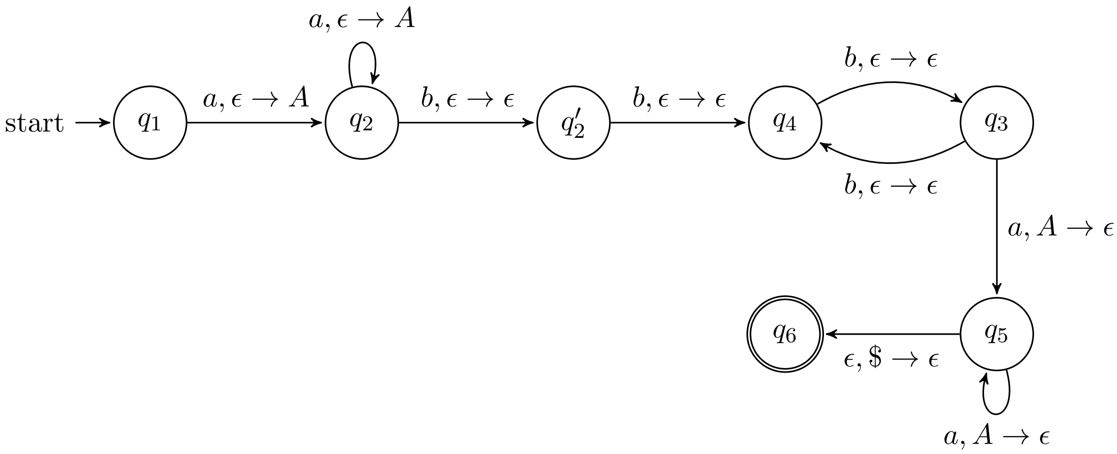
Theoretical Computer Science - Bridging Course
Summer Term 2018
Exercise Sheet 4

for getting feedback submit (electronically) before the start of the tutorial on 19th of November 2018.

Exercise 1: Constructing Pushdown Automata *(6 Points)*

Consider the language $L = \{a^n b^{2m} b a^n \mid m, n > 0\}$ over the alphabet $\Sigma = \{a, b\}$.
Construct a PDA $\mathcal{A}$ with $L(\mathcal{A}) = L$.

Sample Solution



The formal definition of the automaton is implicitly given.

Exercise 2: Chomsky Normal Form *(5 Points)*

Use the algorithm from the lecture to give a grammar in Chomsky Normal Form that generates the same language as the grammar $G = (V; \Sigma; R; S)$ with $V = \{S; X; Y\}, \Sigma = \{a, ab, c\}$, and R being the following set of rules:

$$S \to XY$$
$$X \to abb|aXb|\epsilon$$
$$Y \to c|cY$$

Sample Solution

$$S_0 \to S$$
$$S \to XY$$
$$X \to abb|aXb|\epsilon$$
$$Y \to c|cY$$

$$S_0 \to S$$
$$S \to XY|Y$$
$$X \to abb|aXb|ab$$
$$Y \to c|cY$$
$$S_0 \to S$$
$$S \to XY|c|cY$$
$$X \to abb|aXb|ab$$
$$Y \to c|cY$$

$$S_0 \to S$$
$$S \to XY|c|cY$$
$$X \to aX_1|aX_2|ab$$
$$X_1 \to bb$$
$$X_2 \to Xb$$
$$Y \to c|cY$$

$$S \to XY|c|CY$$
$$X \to AX_1|AX_2|AB$$
$$X_1 \to BB$$
$$X_2 \to XB$$
$$Y \to c|CY$$
$$A \to a$$
$$B \to b$$
$$C \to c$$

Exercise 3: Context-Free Languages and Set Operations *(3+3 Points)*

(a) Show that context-free languages are not closed under taking intersections (i.e., the intersection of two context-free languages is not necessarily context free).
Hint: You can use that the language $\{a^i b^i c^i | i \geq 0\}$ *is not context-free.*

(b) Show that context-free languages are not closed under taking complements.
Hint: You can use DeMorgan's law and the fact that the set of context-free languages is closed under performing union operations.

Sample Solution

(a) Assume context-free languages are closed under intersection operation. We prove the claim by contradiction. Consider the following languages: $L_1 = \{a^i b^i c^j | i, j \geq 0\}$, $L_2 = \{a^i b^j c^j | i, j \geq 0\}$, and $L_3 = L_1 \cap L_2 = \{a^i b^i c^i | i \geq 0\}$. It is easy to prove that L_1 and L_2 are both context-free languages (as you can easily derive the corresponding context-free grammar). According to our assumption, we know L_3 will be context-free language as well. However, we know by the pumping lemma that L_3 is not context-free. Hence, we have a contradiction, which implies the assumption is wrong. Therefore, we have proved the claim.

(b) Assume context-free languages are closed under taking complement operation. We prove the claim by contradiction. Let L_1 and L_2 both be context-free languages. According to the assumption, we know $\overline{L_1}$ and $\overline{L_2}$ must both be context-free languages as well. Since context-free languages are closed under union operation. We know $\overline{L_1} \cup \overline{L_2}$ is context-free too. Now, according to the DeMorgan's law, we know $\overline{\overline{L_1} \cup \overline{L_2}} = L_1 \cap L_2$. As a result, we can further conclude $L_1 \cap L_2$ is context-free. Since L_1 and L_2 are arbitrary context-free languages, this implies context-free languages are closed under intersection operation. However, from part a), we know this is not true. Therefore, we have a contradiction, and have hence proved the original claim.