University of Freiburg
Department of Computer Science
Prof. Dr. F. Kuhn

Exam Theoretical Computer Science - Bridging Course

Monday, March 4, 2019, 14:00-15:30

Name: .....

Matriculation No.: .....

Signature: .....

Do not open or turn until told so by the supervisor!

- Write your **name** and **matriculation number** on this page and sign the document!
- Write your name on **all sheets**!
- Your **signature** confirms that you feel physically and mentally able to write the exam and that you have answered all questions without any help.
- Write legibly and only use a pen (ink or ball point). Do **not use red**! Do **not use a pencil**!
- This is an **open book exam** therefore printed or hand-written material is allowed.
- However, **no electronic devices** are allowed.
- There are **eight tasks** (with several sub-tasks each) and there is a **total of 90 points**.
- **35 points are sufficient** in order to pass the exam. **70 points** are sufficient to get the best mark.
- Only **one solution per task** is considered! Make sure to strike out alternative solutions, otherwise the one yielding the minimal number of points is considered.
- **Detailed steps** might help you to get more points in case your final result is incorrect.
- The keywords **Show...** or **Prove...** indicate that you need to prove or explain your answer carefully.
- The keywords **Give...** or **State...** indicate that you only need to provide a plain answer.
- You may use information given in a **Hint** without explaining them.
- **Read each task thoroughly** and make sure you understand what is expected from you.
- **Raise your hand** if you have a question regarding the formulation of a task.
- **Use the space below each task and the back of the sheet for your solution.** The last two sheets of this exam are blank and can be used for solutions. If you need additional sheets, raise your hand.

Question	1	2	3	4	5	6	7	**Total**
Points								
Maximum	10	19	10	14	10	12	15	90

Task 1: Basic Mathematical Skills (10 Points)

1. A *tree* is a simple, connected graph without cycles. Show that a tree with n nodes has $n-1$ edges. *(5 Points)*

   Hint: You may use that a tree has at least one leaf, i.e., a node of degree one.

2. Let $T_1 = (V, E_1)$, $T_2 = (V, E_2)$, ..., $T_k = (V, E_k)$ be k trees on the same set of vertices V of size n (assume that n is even). Let $G = (V, E_1 \cup E_2 \cup \ldots \cup E_k)$ be the union of these trees. Show that more than half of the nodes of G have degree at most $4k$ in G. *(5 Points)*

Task 2: Regular Languages (19 Points)

1. Let $\Sigma = \{a, b, \ldots, z\}$ be the set of letters from the English alphabet. Let L be the language over Σ consisting of all words that appear in the book "Harry Potter and the Chamber of Secrets". Is L regular? Explain your answer in one sentence. *(2 Points)*

2. Let $\Sigma = \{a, b\}$. Let L_1 be the language defined by the regular expression $a^*b^*a^*$ and L_2 the language defined by a^*b^*b. *(7 Points)*

   Draw a DFA for L_1, L_2, and $L_1 \setminus L_2 := \{w \in \Sigma^* \mid w \in L_1 \text{ and } w \notin L_2\}$.

3. Show that if L and L' are regular languages over some alphabet Σ, then also $L \setminus L'$ is regular. *(3 Points)*

4. Use the pumping lemma to show that $L = \{w \in \{a, b\}^* \mid w \text{ contains more } a\text{'s than } b\text{'s}\}$ is not regular. *(7 Points)*

Task 3: Context-Free Languages (10 Points)

Give a context-free grammar that generates the language

$$\{a^i b^j c^k \mid i = j \text{ or } j = k \text{ where } i, j, k \geq 0\} .$$

Hint: It is maybe helpful to remember that context-free languages are closed under union.

Task 4: Decidability (14 Points)

1. Show that $A = \{\langle R, S\rangle \mid R \text{ and } S \text{ are regular expressions and } L(R) \subseteq L(S)\}$ is decidable. *(8 Points)*

2. Show that $EQ_{TM} = \{\langle M_1, M_2\rangle \mid M_1, M_2 \text{ are Turing Machines and } L(M_1) = L(M_2)\}$ is undecidable. *(6 Points)*

   Hint: You may use that $E_{TM} = \{\langle M\rangle \mid M$ is a Turing Machine and $L(M) = \emptyset\}$ is undecidable.

Task 5: $\mathcal{O}$ - Notation (10 Points)

State whether the following claims are true or false *(1 point each)*. Then **prove or disprove** the claim. Use the definition of the $\mathcal{O}$-notation.

1. $(\ln n)^2 \in \mathcal{O}\left(\ln(n^2)\right)$ *(1+4 Points)*
2. $3n^2 + 8n \in \mathcal{O}\left(n^2\right)$ *(1+4 Points)*

Task 6: Complexity (12 Points)

Given a set U of n elements ('universe') and a collection $S \subseteq \mathcal{P}(U)$ of subsets of U, a selection $C_1, \ldots, C_k \in S$ of k sets is called a *set cover* of (U, S) of size k if $C_1 \cup \ldots \cup C_k = U$.

Show that the problem

$$\textsc{SetCover} := \{\langle U, S, k\rangle \mid U \text{ is a set, } S \subseteq \mathcal{P}(U) \text{ and there is a set cover of } (U, S) \text{ of size } k\}$$

is NP-complete.

You may use that

$$\textsc{DominatingSet} = \{\langle G, k\rangle \mid G \text{ has a dominating set with } k \text{ nodes}\}.$$

is NP-complete. A subset of the nodes of a graph G is a *dominating set* if every other node of G is adjacent to some node in the subset.

Task 7: Logic (15 Points)

1. Consider the following propositional formula

$$\psi := (x \vee y \to \bot) \wedge (z \to x \wedge w) \wedge (y \vee z).$$

Either find a satisfying assignment for ψ or use the resolution calculus to show that ψ is unsatisfiable. *(9 Points)*

2. Consider the following first order logical formulae

$$\begin{aligned}
\varphi_1 &:= \forall x \ \neg R(x, x) \\
\varphi_2 &:= \forall x \, \forall y \ (x \neq y \to R(x, y) \vee R(y, x)) \\
\varphi_3 &:= \exists x \, \forall y \ (x \neq y \to R(x, y))
\end{aligned}$$

where x, y are variable symbols and R is a binary predicate. Give an interpretation

   (a) I_1 which is a model of $\varphi_1 \wedge \varphi_2 \wedge \varphi_3$. *(3 Points)*

   (b) I_2 which is a model of $\varphi_1 \wedge \varphi_2 \wedge \neg\varphi_3$. *(3 Points)*

Remark: No proof required.