

Theoretical Computer Science - Bridging Course

Winter Term 2019/2020

Exercise Sheet 3

for getting feedback submit electronically by 12:15 am, Monday, November 11, 2019

Exercise 1: Regular Expressions

(6 Points)

Give a regular expression for each of the following three languages.

- (a) $L_1 = \{w_1w_2w_3 \mid w_1, w_2, w_3 \in \{a, b, c\}^*, w_1 \text{ contains no } a, w_2 \text{ contains no } b, w_3 \text{ contains no } c\}$
- (b) $L_2 \subseteq \{a, b\}^*$ is the language of all words that do not have any of the words in $\{aaa, aaaa, \dots\}$ as a consecutive substring.
- (c) L_3 is the language, over alphabet $\{a, b\}$, of all strings not ending with aa .

Exercise 2: The Pumping Lemma: Sufficiency or Necessity? (4 Points)

Consider the language $L = \{c^m a^n b^n \mid m, n \geq 0\} \cup \{a, b\}^*$ over the alphabet $\Sigma = \{a, b, c\}$.

- (a) Describe in words (not using the pumping lemma), why L cannot be a regular language.
- (b) Show that the property described in the Pumping Lemma is a necessary condition for regularity but not sufficient for regularity.

Hint: Use L as counter example, i.e., show that it can be 'pumped' (in the sense of the pumping lemma), but is still not regular.

Exercise 3: Application of The Pumping Lemma

(6 Points)

Show that the following languages over the alphabet $\Sigma = \{a, b\}$ are not regular:

- (a) $L = \{a^m \mid m \text{ is a square number}\}$ (m being a square number means that $m = n^2$ for some non negative integer n)

Hint: Use the Pumping Lemma.

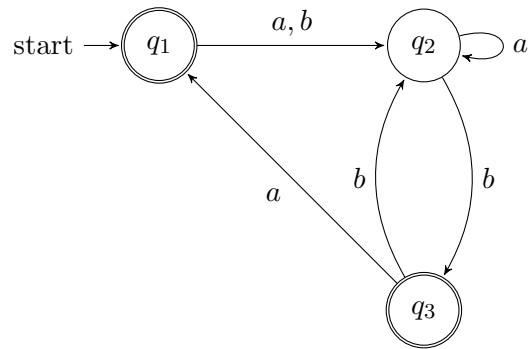
- (b) $L = \{a^m b^n \mid m \neq n\}$

Hint: Have a look at the languages $\{a^n b^n \mid n \in \mathbb{N}\}$ and $a^ b^*$ and use the fact that the class of regular languages is closed under intersection, complement, concatenation and the Kleene star.*

Exercise 4: NFAs to Regular Expressions

(4 Points)

Consider the following NFA:



Give the regular expression defining the language recognized by this NFA by *stepwise* converting it into an equivalent GNFA with only two nodes.