University of Freiburg
Dept. of Computer Science
Prof. Dr. F. Kuhn

Algorithms and Data Structures
Winter Term 2021/2022
Exercise Sheet 5

Exercise 1: Bad Hash Functions

Let m be the size of a hash table and $M \gg m$ the largest possible key of the elements we want to store in the table. The following "hash functions" are poorly chosen. Explain for each function why it is not a suitable hash function.

(a) $h : x \mapsto \lfloor \frac{x}{m} \rfloor \bmod m$

(b) $h : x \mapsto (2x + 1) \bmod m$ (m even).

(c) $h : x \mapsto (x \bmod m) + \lfloor \frac{m}{x+1} \rfloor$

(d) For each calculation of the hash value of x one chooses for $h(x)$ a uniform random number from $\{0, \ldots, m-1\}$

(e) For a set of "good" hash functions $h_1, \ldots, h_\ell$ with $\ell \in \Theta(\log m)$, we first compute $h_1(x)$, then $h_2(h_1(x))$ etc. until $h_\ell(h_{\ell-1}(\ldots h_1(x)) \ldots)$. That is, the function is $h : k \mapsto h_\ell(h_{\ell-1}(\ldots h_1(x)) \ldots)$

Exercise 2: (No) Families of Universal Hash Functions

Let $\mathcal{S} = \{0, \ldots, M-1\}$ and $\mathcal{H}_1 := \{h : x \mapsto a \cdot x^2 \bmod m \mid a \in \mathcal{S}\}$. Show that H_1 is not c-univeral for *constant* $c \geq 1$ (that is c is fixed and must not depend on m).