Exercise 1: Sort Functions by Asymptotic Growth  \( (8 \text{ Points}) \)

Give a sequence of the following functions sorted by asymptotic growth, i.e., for consecutive functions \( g, f \) in your sequence, it should hold \( g \in \mathcal{O}(f) \). Write “\( g \sim f \)” between two functions in the sequence if \( f \in \mathcal{O}(g) \) and \( g \in \mathcal{O}(f) \).

\[
\begin{align*}
\log_2(n!) & \quad \sqrt{n} & \quad \sqrt{n^3} & \quad \log_2(n^2) \\
n^2 + n \log_2(n^2) & \quad 3^n & \quad n^{\log_2 n} & \quad (\log_2 n)^2 \\
\log_{10} n & \quad 10^{100} \cdot n & \quad n! & \quad n \log_2 n \\
n \cdot 2^n & \quad n^n & \quad \sqrt{\log_2 n} & \quad n^2
\end{align*}
\]

Exercise 2: Maximum Rectangle in a Histogram  \( (12 \text{ Points}) \)

Consider a sequence \( h_1, \ldots, h_n \) of positive, integer numbers. This sequence represents a histogram \( H \) consisting of \( n \) horizontally aligned bars each of width 1, where \( h_i \) represents the height of the \( i^{th} \) bar. The goal is to find a rectangle of maximum area completely within \( H \) (i.e., within the union of bars).

(a) Describe an algorithm that computes a maximum area rectangle in \( H \) in time \( \mathcal{O}(n^2) \).

(b) Describe an algorithm that computes a maximum area rectangle that is within \( H \) and also intersects the \( i^{th} \) bar in time \( \mathcal{O}(n) \) and prove the running time.

\textit{Remark: correct solutions in } o(n^2) \text{ grant partial points.}

(c) Give an algorithm that uses the divide and conquer principle to compute a maximum area rectangle in \( H \) in time \( \mathcal{O}(n \log n) \) and prove the running time.

\textit{Remark: you can use part (b), even if you did not succeed.}