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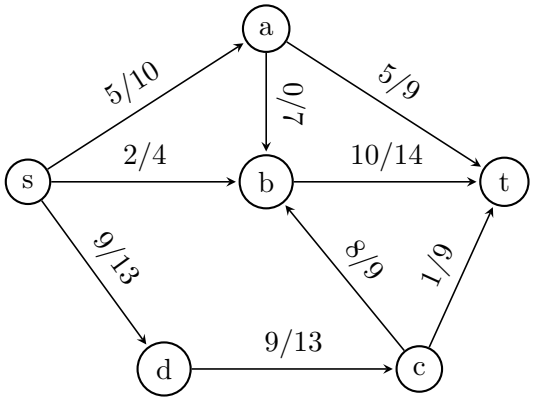
Algorithm Theory
Exercise Sheet 8

Due: Friday, 15th of December 2023, 10:00 am

Exercise 1: Ford-Fulkerson Algorithm *(6 Points)*

Consider the following flow network, where for each edge, the capacity (second number) and a current flow value (first number) are given. Solve the maximum flow problem on the network by using the Ford-Fulkerson variant that always picks a best possible augmenting path (an augmenting path that improves the current flow from s to t by as much as possible) in every iteration. Give intermediate results, i.e., draw the residual graph with all the residual capacities in every iteration.



Exercise 2: Seating Arrangement *(4 Points)*

A group of students goes out to eat dinner together. To increase social interaction, they would like to sit at tables such that no two students from the same faculty are at the same table. For that purpose assume there are students from x different faculties while $n_1, n_2, ..., n_x$ describe the affiliated number of students from these x faculties. Also assume that there are y tables available while r_j students can take place on the j-th table.
Let us define the *seating arrangement* as the decision problem returning `true` if one can distribute the students from same faculties to different tables and `false` otherwise. Formulate this *seating arrangement* problem as a *maximum flow* problem and write down the condition that should hold whenever the original decision problem returns `true`. Further, give the runtime it takes to solve the corresponding flow problem in terms of x, y, n_i and r_j for all $1 \leq i \leq x$ and $1 \leq j \leq y$.

Exercise 3: Smallest Minimum Cut *(10 Points)*

Let $G = (V, E)$ be a flow network with *integer capacities* $c_e \geq 0$ for all $e \in E$. Give a new flow network $G' = (V, E)$ (that has the same nodes and edges as G) with *integer capacities* $c'_e \geq 0$ such that any minimum cut in G' is a minimum cut in G with the smallest number of edges (of all minimum cuts in G). Proof your statement!
Hint: Consider capacities $c'_e := c_e + 1$. How does the capacity of a cut change by this choice? Does this already solve the task or do you need to adjust it?