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Algorithm Theory
Exercise Sheet 9

Due: Friday, 22nd of December 2023, 10:00 am

Exercise 1: Mining Operations   (12 Points)

The FY Corporation has decided to begin mining operations on a remote island. They have done preliminary tests, so they know what kind of jobs (operations) they can do. They know that there are n operations available, all of them has p_i, $\forall i = 1, \ldots, n$ value (this can be negative). They also know that some operations are prerequisites for other operations, e.g., the job i has to be completed before j. It can also happen that an operation has many prerequisites. Your task is to find a set of jobs S that are prerequisite complete, meaning every operation includes every other operation that is a prerequisite for it, in the set S, such that the sum of the p_i for these jobs is maximum. Give a polynomial-time algorithm that achieves this solution.

Exercise 2: Matching in Bipartite Graphs   (2+2+4 Points)

(a) Prove that in a k-regular (every vertex has degree k) bipartite graph there exists a perfect matching.

(b) Prove that in a k-regular bipartite graph the edge set can be partitioned into k perfect matchings.

(c) Prove that in a bipartite graph there exists a matching which covers every vertex that has maximum degree.