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Algorithm Theory
Sample Solution Exercise Sheet 15

Due: Friday, 16th of February 2024, 10:00 am

Exercise 1: Maximum Sparse Subgraph *(8 Bonus Points)*

Let $G = (V, E)$ be a simple graph, we define a 2-sparse edge set $F \subseteq E$ to be the subset of edges such that the subgraph $H = (V, F)$ induced by the edges has maximum degree at most 2. We call F a maximum 2-sparse edge set if there is no 2-sparse edge set F' of larger size.
Assume that we are given the edges in online fashion. Observe the greedy algorithm that always adds the next incoming edge $e = \{u, v\}$ to F as long as this choice does not increase the degree of u or v to more than 2. Show that this algorithm has competitive ratio 2. Further, show that this is tight, i.e., there is a graph such that if the edges come in some specific order, the algorithm has exactly half as many edges as to optimal solution.

Exercise 2: Parallel Parentheses *(12 Bonus Points)*

You are given a string S consisting of opening and closing parentheses. The expression S is supposed to be a string with **balanced parentheses** if each opening parenthesis has a corresponding closing one and the pairs of parentheses are properly nested. For example, consider the expressions (()()) as a correctly balanced string of parentheses and ())(as an incorrect one.

a) First, provide a (sequential) linear-time algorithm to determine whether S is balanced. *(4 Points)*

b) Now devise a parallel algorithm to check if the string S is balanced. Like in the lecture assume that we are given p processors. What is the asymptotic runtime T_p? *(8 Points)*