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Algorithms and Datastructures
Winter Term 2024
Exercise Sheet 4

Due: Wednesday, November 27th, 2pm

Exercise 1: Hashing with Open Addressing *(5 Points)*

Let $\mathcal{H}$ be a hash table of size $m = 13$ and let $h_1, h_2, h_3 : \mathbb{N}_0 \mapsto \{0, ..., m-1\}$ be hash functions defined as follows[1]:

- $h_1(k) := \overline{k} \mod m$
- $h_2(k) := 3 \cdot k \mod m$
- $h_3(k) := k + 1 \mod m$

Add the keys $23, 12, 75, 945, 30, 99, 345$ (in that order) into the initaly empty hash table $\mathcal{H}$. Solve conflicts as follows:

a) Linear Probing using hash function h_1. *(2 Points)*

b) Use Double Hashing using hash functions h_2 and h_3. *(3 Points)*

Write down every intermediate step!

Exercise 2: Hashing with Chaining *(5 Points)*

Given a Hash Table of size m and an arbitrary hash function $h : S \mapsto \{0, ..., m-1\}$. Let S be a set of at least $y \cdot m$ elements, so $|S| \geq y \cdot m$.

a) Show that S has a subset Y of at least y elements (hence $|Y| \geq y$) such that $h(x_1) = h(x_2)$ for all $x_1, x_2 \in Y$. *(4 Points)*

b) What does the result of *a)* tells us about the Worst-Case runtime of *"find"* in a hash table with Chaining (if the table is filled with all the elements of S before we call *"find"*)? *(1 Point)*

Exercise 3: Application of Hashtables *(10 Points)*

Consider the following algorithm:

Algorithm 1 `algorithm` ▷ Input: Array A of length n with integer entries

```
for i = 1 to n − 1 do
    for j = 0 to i − 1 do
        for k = 0 to n − 1 do
            if |A[i] − A[j]| = A[k] then
                return true
return false
```

[1]We define the digit sum of k by $\overline{k}$.

(a) Describe what `algorithm` computes and analyse its asymptotical runtime. *(3 Points)*
Hint: The difference $|A[i] - A[j]|$ may become arbitrarily large.

(b) Describe a different algorithm $\mathcal{B}$ for this problem (i.e., $\mathcal{B}(A) = \texttt{algorithm}(A)$ for each input A) which uses hashing and takes time $\mathcal{O}(n^2)$ (with proof). *(3 Points)*

Hint: You may assume that inserting and finding keys in a hash table needs $\mathcal{O}(1)$ if $\alpha = \mathcal{O}(1)$ (α is the load of the table).

(c) Describe another algorithm for this problem without using hashing which takes time $\mathcal{O}(n^2 \log n)$ (with proof). *(4 Points)*