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Algorithm Theory
Bonus Sheet

Due: Friday, 17th of January 2024, 10:00 am

Bonus Points: The points achievable in this exercise sheet will not increase the threshold of points needed.

Exercise 1: Edge-Coloring *(8 Bonus Points)*

We define a *proper edge-coloring* of a graph as an assignment of 'colors' to the edges of the graph so that no two incident edges have the same color. For example, if the graph is a path, there is a proper edge-coloring with two colors as one can color the edges in alternating fashion from left to right.
For some $d \in \mathbb{N}$, we call a simple undirected graph d-regular if each node has exactly d neighbors. Let $G_d = (A \dot{\cup} B, E)$ be a d-regular bipartite graph.

(a) Show that there exists a perfect matching in G_d. *(4 Points)*

(b) Show that there exists a *proper edge-coloring* with colors $\in \{1, \ldots, d\}$ in G_d. *(4 Points)*

Exercise 2: Chess tournament *(6 Bonus Points)*

Assume that there are n chess players $1, \ldots, n$ that you need to pair up for playing against each other in a chess tournament.
There are some players who must play their next game with the white pieces and there are some players who must play their next game with the black pieces. There are also players for whom it does not matter if they play with the white or the black pieces.
In addition, each player i has a rating value r_i, which is a positive integer.
Each chess game in the tournament must be played between exactly two players: one playing with the white pieces and the other with the black pieces. Further, each player should play in at most one game. Additionally, to ensure balanced games, the absolute difference in rating between the two players in a game must be smaller than 100.

a) Describe a polynomial-time algorithm to determine a largest possible set of chess games that can be arranged with the available n players. You can use algorithms from the lecture as a black box. *(3 Points)*

b) Assume that we make the (strange) requirement that the absolute difference in rating between the two players of a game must be an odd number < 100? Argue why the problem of determining a maximum set of possible chess games now becomes easier! *(3 Points)*