



# Theoretical Computer Science - Bridging Course

## Exercise Sheet 4

**Due:** Tuesday, 19th of November 2024, 12:00 pm

### Exercise 1: CFGs and PDAs

*(5+3 Points)*

Give a context free grammar for each of the following languages.

- $L_1 = \{a^i b^j \mid 0 < i \leq j\}$
  - $L_2 = \{a^{2n} b^n \mid n > 0\}$
  - $L_3 = \{a^* w c^k \mid w \in \{a, b\}^*, \text{ and } k \text{ is the number of } a\text{'s in } w\}$
2. Create a pushdown automaton that accepts languages  $L_2$  and  $L_3$ .

### Exercise 2: Proving NonCFL

*(4+4 Points)*

Use the Pumping Lemma to show that the following languages are not CFL.

1.  $L_4 = \{a^n b a^{2n} b a^{3n} \mid n \geq 0\}$
2.  $L_5 = \{a^i b^j c^k \mid i < j \text{ and } i < k\}$

*Bonus:*  $L_6 = \{a^m \mid m \text{ is a prime}\}$

*NB:* If you wish you can try first and prove it nonregular using the Pumping Lemma for regular languages and the same idea should be extended to CFLs.

### Exercise 3: Closure in CFL

*(2+2 Points)*

1. Show that the context-free languages are closed under union, concatenation and Kleene star.  
*Hint:* try to prove that the context-free languages are closed under the above operators via creating the appropriate grammars.
2. Knowing that  $L_7 = \{a^i b^j c^k \mid i < j\}$  is a context free language, are context free languages closed under intersection?  
*Hint:* Use the fact that  $L_5$  is not a context free language.