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Theoretical Computer Science - Bridging Course
Exercise Sheet 5

Due: Tuesday, 26th of November 2024, 12:00 pm

Exercise 1: Operation Shift *(3+3 Points)*

Consider a Turing machine $\mathcal{M}$ that is given an arbitrary input string over alphabet $\Sigma = \{1, 2, \dots, n\}$ on its input tape. We would like $\mathcal{M}$ to insert an empty cell, i.e., $\sqcup$, at the beginning of the tape without removing any symbol on the tape. As an example, the Turing machine is supposed to change the input tape of the form $\langle 2, 4, 4, 6, 1, 8, 4, \sqcup, \sqcup, \dots \rangle$ to $\langle \sqcup, 2, 4, 4, 6, 1, 8, 4, \sqcup, \sqcup, \dots \rangle$. Although this operation is not explicitly defined for a Turing machine, one can consider such an operation as shifting the whole string one cell to the right on the input tape.

(a) Give a formal definition of $\mathcal{M}$ to perform the desired operation such that $\mathcal{M}$ recognizes the language Σ^*.

(b) For $n = 2$, i.e., $\Sigma = \{1, 2\}$, draw the state diagram of your constructed Turing machine.

Exercise 2: Constructing TMs (Part 1) *(3+3 Points)*

1. Consider alphabet $A = \{1, 2, \dots, 9\}$. We call a string S over A a *blue* string, if and only if the string consisting of the odd-positioned symbols in S is the reverse of the string consisting of the even-positioned symbols in S. For example $S = 14233241$ is a blue string since the substring of the odd-positioned symbols is 1234 which is the reverse of the substring of the even-positioned symbols, i.e., 4321.

   Design a Turing machine which accepts all blue strings over A. You do not need to provide a formal description of the Turing machine but your description has to be detailed enough to explain every possible step of a computation.

2. Construct a Turing machine that decides on the languages
   $C_1 = \{a^i b^j c^k | i - j = k \text{ and } i, j, k \geq 1\}$ and $C_2 = \{a^i b^j c^k | i \times j = k \text{ and } i, j, k \geq 1\}$.

Exercise 3: Constructing TMs (Part 2) *(3+2+2+1 Points)*

Let $\Sigma = \{0, 1\}$. For a string $s = s_1 s_2 \dots s_n$ with $s_i \in \Sigma$, let $s^R = s_n s_{n-1} \dots s_1$ be the *reversed* string. *Palindromes* are strings s for which $s = s^R$. Then $L = \{sas^R \mid s \in \Sigma^*, a \in \Sigma \cup \{\varepsilon\}\}$ is the language of all palindromes over Σ.

(a) Give a state diagram of a Turing machine recognizing L.

(b) Give the maximum number (or a close upper bound for the number) of head movements your Turing machine makes until it halts, if started with an input string $s \in \Sigma^*$ of length $|s| = n$ on its tape.

(c) Describe (informally) the behavior of a 2-tape Turing machine which recognizes L and uses significantly fewer head movements on long inputs than your 1-tape Turing machine.

(d) Give the maximum number (or a close upper bound for the number) of head movements your Turing machine makes on any of the two tapes until it halts, if started with an input string $s \in \Sigma^*$ of length $|s| = n$ on the first tape.