



# Theoretical Computer Science - Bridging Course

## Exercise Sheet 11

Due: Tuesday, 21st of January 2025, 12:00 pm

### Exercise 1: Construct Formulae

(1+1+1 Points)

Let  $\mathcal{S} = \langle \{x, y, z\}, \emptyset, \emptyset, \{R\} \rangle$  be a signature. Translate the following sentences of first order formula over  $\mathcal{S}$  into idiomatic English. Use  $R(x, y)$  as statement 'x is a part of y'.

- (a)  $\exists x \forall y R(x, y)$ .
- (b)  $\exists y \forall x R(x, y)$ .
- (c)  $\forall x \forall y \exists z (R(x, z) \wedge R(y, z))$

### Exercise 2: FOL: Is it a model?

(2+3+3 Points)

Consider the following **first order** formulae

$$\begin{aligned}\varphi_1 &:= \forall x R(x, x) \\ \varphi_2 &:= \forall x \forall y R(x, y) \rightarrow (\exists z R(x, z) \wedge R(z, y)) \\ \varphi_3 &:= \exists x \exists y (\neg R(x, y) \wedge \neg R(y, x))\end{aligned}$$

over signature  $\mathcal{S}$  where  $x, y, z$  are variable symbols and  $R$  is a binary predicate. Give an interpretation

- (a)  $I_1$  which is a **model** of  $\varphi_1 \wedge \varphi_2$ .
- (b)  $I_2$  which is **no model** of  $\varphi_1 \wedge \varphi_2 \wedge \varphi_3$ .
- (c)  $I_3$  which is a **model** of  $\varphi_1 \wedge \varphi_2 \wedge \varphi_3$ .

### Exercise 3: FOL: Entailment

(3+3+3 Points)

Let  $\varphi, \psi$  be first order formulae over signature  $\mathcal{S}$ . Similar to propositional logic, in predicate logic we write  $\varphi \models \psi$  if every model of  $\varphi$  is also a model for  $\psi$ . We write  $\varphi \equiv \psi$  if both  $\varphi \models \psi$  and  $\psi \models \varphi$ . A *knowledge base*  $KB$  is a set of formulae. A model of  $KB$  is model for all formulae in  $KB$ . We write  $KB \models \varphi$  if all models of  $KB$  are models of  $\varphi$ . Show or disprove the following entailments.

- (a)  $(\exists x R(x)) \wedge (\exists x P(x)) \wedge (\exists x T(x)) \models \exists x (R(x) \wedge P(x) \wedge T(x))$ .
- (b)  $(\forall x \forall y f(x, y) \doteq f(y, x)) \wedge (\forall x f(x, \mathbf{c}) \doteq x) \models \forall x f(\mathbf{c}, x) \doteq x$ .
- (c)  $(\forall x R(x, x)) \wedge (\forall x \forall y R(x, y) \wedge R(y, x) \rightarrow x \doteq y) \wedge (\forall x \forall y \forall z R(x, y) \wedge R(y, z) \rightarrow R(x, z))$   
 $\models \forall x \forall y R(x, y) \vee R(y, x)$ .

*Hint: Consider order relations. E.g.,  $a \leq b$  ( $a$  less-equal  $b$ ) and  $a|b$  ( $a$  divides  $b$ ).*