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Dept. of Computer Science
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Algorithm Theory
Exercise Sheet 1

Due: Friday, 24th of October 2025, 10:00 am

Exercise 1: General Instructions *(0 Points)*

- Work on this exercise sheet in your group.
- **Everyone** has to submit the solution separately via Daphne.
  ⇒ For that, create a new folder named `exercise-01` and place your file(s) inside.
- Make sure each group members name is visible on your solution sheet (preferably Name and RZ-Account).
- Please submit the solution as PDF file (no .tex, .txt or .doc needed). Handwritten scans are okay, but make sure they are readable. The recommendation is to use LaTeX for submissions.
- If you have any kind of question regarding the exercise sheet, use the Zulip forum to ask.

Exercise 2: Recurrence Relation *(10 Points)*

You are given the following recurrence relation:

$$T(n) \leq 2 \cdot T\left(\frac{n}{2}\right) + \sqrt{2 \cdot n}, \quad T(1) = 1.$$

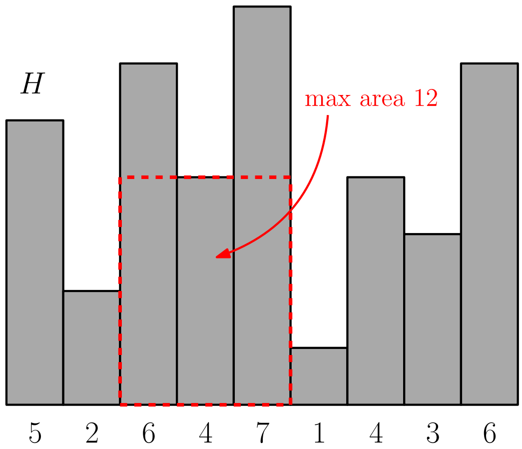
a) *Guess* a (closed-form) function $f(n)$, such that for all $n \geq 1$ it holds that $T(n) \leq f(n)$. Try to choose $f(n)$ as (asymptotically) *tight* upper bound of $T(n)$ as possible. *(4 Points)*
Remark: You should not just guess out of the box, instead, the idea is to find a sophisticated guess by repeatedly substituting the recurrence relation into itself.

b) Use *induction* to show that your guess is correct. *(4 Points)*
Remark: You can assume that n equals 2^j for some $j \in \mathbb{N}$.

c) So what is $T(n)$ in $\mathcal{O}$-notation due to your result? Can you achieve the same result using the Master Theorem? If so, explain which of the 3 cases you use, otherwise explain why it does not work. *(2 Points)*

Exercise 3: Maximum Rectangle in a Histogram *(10 Points)*

Consider a sequence $h_1, \dots, h_n$ of positive, integer numbers. This sequence represents a histogram H consisting of n horizontally aligned bars each of width 1, where h_i represents the height of the i^{th} bar. The goal is to find a rectangle of maximum area completely within H (i.e., within the union of subsequent bars).



(a) Describe an algorithm that computes a maximum area rectangle in H in time $\mathcal{O}(n^2)$. *(2 Points)*

(b) Describe an algorithm that computes a maximum area rectangle that is within H and also intersects the i^{th} bar in time $\mathcal{O}(n)$ and prove the running time. Also argue why your algorithm is correct, i.e., why there can not exist a rectangle intersecting i with a strictly larger area than the outcome of your algorithm. *(5 Points)*

Remark: Correct solutions in $o(n^2)$ grant partial points.

(c) Give an algorithm that uses the *divide and conquer* principle to compute a maximum area rectangle in H in time $\mathcal{O}(n \log n)$ and prove the running time. *(3 Points)*

Remark: You can use part (b), even if you did not succeed.