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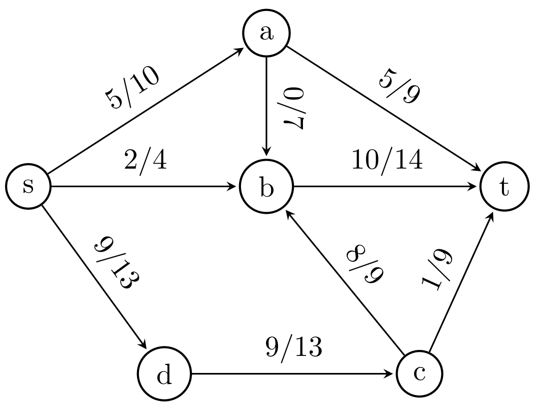
Algorithm Theory
Exercise Sheet 8

Due: Friday, 12th of December 2025, 10:00 am

Exercise 1: Ford-Fulkerson Algorithm *(4 Points)*

Consider the following flow network, where for each edge, the capacity (second number) and a current flow value (first number) are given. Solve the maximum flow problem on the network by using the Ford-Fulkerson variant that always picks a best possible augmenting path (an augmenting path that improves the current flow from s to t by as much as possible) in every iteration. Give intermediate results, i.e., draw the residual graph with all the residual capacities in every iteration.



Exercise 2: Escape Problem *(6 Points)*

An $n \times n$ grid is an undirected graph consisting of n rows and n columns of vertices. We denote the vertex in the ith row and the jth column by (i, j). All vertices in a grid have exactly four neighbors, except for the boundary vertices, which are the points (i, j) for which $i = 1, i = n, j = 1$, or $j = n$. Given $\ell \leq n^2$ starting points $(x_1, y_1), (x_2, y_2), \ldots, (x_\ell, y_\ell)$ in the grid, the escape problem is to determine whether or not there are ℓ vertex-disjoint paths from the starting points to any ℓ different points on the boundary. Give an algorithm that solves the escape problem in polynomial time. Check the figure for an example.

Exercise 3: Smallest Minimum Cut *(10 Points)*

Let $G = (V, E)$ be a flow network with *integer capacities* $c_e \geq 0$ for all $e \in E$. Give a new flow network $G' = (V, E)$ (that has the same nodes and edges as G) with *integer capacities* $c'_e \geq 0$ such that any minimum cut in G' is a minimum cut in G with the smallest number of edges (of all minimum cuts in G). Proof your statement!
Hint: Consider capacities $c'_e := c_e + 1$. How does the capacity of a cut change by this choice? Does this already solve the task or do you need to adjust it?

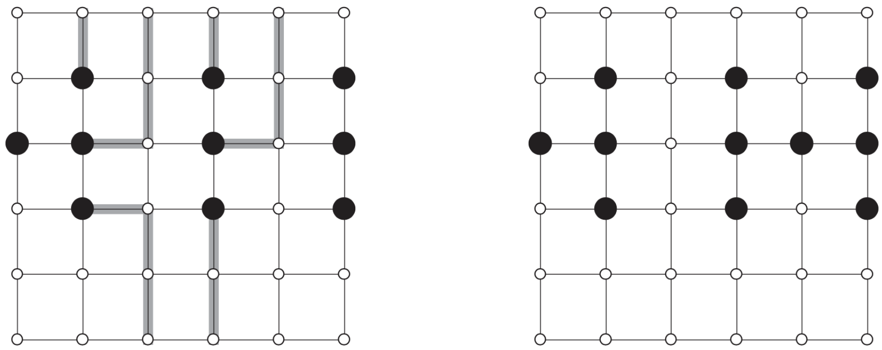


Figure 1: Grids for the escape problem. Starting points are black, and other grid vertices are white. The grid on the left has an escape, shown by shaded paths. The grid on the right has no escape.

Exercise 4: Session Problem: Seating Arrangement *(0 Points)*

A group of students goes out to eat dinner together. To increase social interaction, they would like to sit at tables such that no two students from the same faculty are at the same table. For that purpose assume there are students from x different faculties while $n_1, n_2, ..., n_x$ describe the affiliated number of students from these x faculties. Also assume that there are y tables available while r_j students can take place on the j-th table.

Let us define the *seating arrangement* as the decision problem returning `true` if one can distribute the students from same faculties to different tables and `false` otherwise. Formulate this *seating arrangement* problem as a *maximum flow* problem and write down the condition that should hold whenever the original decision problem returns `true`. Further, give the runtime it takes to solve the corresponding flow problem in terms of x, y, n_i and/or r_j for all $1 \leq i \leq x$ and $1 \leq j \leq y$.

Exercise 5: Session Problem II: Blocked Streets *(0 Points)*

Mr. X has just robbed a bank and is now heading to the harbor. However, the police wants to stop him by closing some streets of the city. As it is expensive to close a street, how can the police decide which streets to close so that there is no route between the bank and the harbor and such that the number of closed streets is minimal?

Assume the city is modeled as an n-node m-edge graph, where the edges represent the streets and the nodes represent the places including the bank and the harbor.