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Distributed Graph Algorithms
Exercise Sheet 8

Motivation. We saw that network decompositions can help us to solve many interesting problems like $\Delta + 1$-coloring, or MIS *deterministically*. Are there other problems for which we can obtain a fast deterministic algorithm using network decomposition?
We will see that network decompositions allow us to solve any problem efficiently that can be solved efficiently in the so called SLOCAL model.

The SLOCAL-Model

In the SLOCAL-Model we are asked to solve a problem Π on a graph $G = (V, E)$. Π is a problem which requires nodes to output values from a set Σ. We process nodes in an arbitrary (adversarially chosen) order. When node v is processed, v can read its r-hop neighborhood and it computes and locally stores its output y and potentially additional information. When reading its r-hop neighborhood, v also reads all the information that has been locally stored by the previously processed nodes there. We call the parameter r the locality of an SLOCAL algorithm.

Exercise 1: $\Delta + 1$-coloring and MIS in SLOCAL

Give locality 1 algorithms for $\Delta + 1$-coloring and MIS in the SLOCAL Model.

Exercise 2: Coloring and SLOCAL

Let Π be a problem that admits a locality T SLOCAL algorithm and assume you are given a distance T-coloring[1] of G with C colors.
Show that Π can be solved in $O(C)$ rounds of the LOCAL model.

Exercise 3: Distance Colorings

What is the minimum number of colors required for a distance T-coloring in a Δ-regular tree? What is the locality of distance T-coloring in SLOCAL?

Exercise 4: Network Decomposition and SLOCAL with locality 1

Let Π be a problem that admits a locality 1 SLOCAL algorithm and assume you are given a (c, d)-network decomposition.
Show that Π can be solved in $O(c \cdot d)$-rounds of the LOCAL model.

[1] In a distance T coloring, is a coloring such that any two vertices within distance T of each other are assigned different colors. Equivalently, two nodes of the same color have distance $> T$, so what is normally understood as a proper coloring is a distance 1 coloring.

Exercise 5: Network Decomposition and SLOCAL with larger localities

Let Π be a problem that admits a locality T SLOCAL algorithm and assume you have a LOCAL algorithm that computes a $(\log n, \log n)$-network decomposition in $O(\log^2 n)$ rounds.
Show that Π can be solved in $O(T \cdot \log^2 n)$ rounds in the LOCAL model.
Hint: Exercise 4 was easy, because of the fact that same colored clusters of the network decomposition are non-adjacent, but if $T > 1$, we need a stronger separation guarrantee, i.e. that clusters are further away from each other.