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Theoretical Computer Science - Bridging Course
Exercise Sheet 5

Due: Tuesday, 18th of November 2025, 12:00 pm

Exercise 1: Proving NonCFL *(5+5 Points)*

Use the Pumping Lemma to show that the following languages are not CFL.

1. $L = \{a^n b a^{2n} b a^{3n} \mid n \geq 0\}$
2. $L = \{a^i b^j c^k \mid i < j \text{ and } i < k\}$

Bonus: $L = \{a^m \mid m \text{ is a prime}\}$
NB: If you wish you can try first and prove it nonregular using the Pumping Lemma for regular languages and the same idea should be extended to CFLs.

Exercise 2: Operation Shift *(3+3 Points)*

Consider a Turing machine $\mathcal{M}$ that is given an arbitrary input string over alphabet $\Sigma = \{1, 2, \ldots, n\}$ on its input tape. We would like $\mathcal{M}$ to insert an empty cell, i.e., $\sqcup$, at the beginning of the tape without removing any symbol on the tape. As an example, the Turing machine is supposed to change the input tape of the form $\langle 2, 4, 4, 6, 1, 8, 4, \sqcup, \sqcup, \ldots \rangle$ to $\langle \sqcup, 2, 4, 4, 6, 1, 8, 4, \sqcup, \sqcup, \ldots \rangle$. Although this operation is not explicitly defined for a Turing machine, one can consider such an operation as shifting the whole string one cell to the right on the input tape.

(a) Give a formal definition of $\mathcal{M}$ to perform the desired operation such that $\mathcal{M}$ recognizes the language Σ^*.

(b) For $n = 2$, i.e., $\Sigma = \{1, 2\}$, draw the state diagram of your constructed Turing machine.

Exercise 3: Constructing a TM *(4 Points)*

Consider alphabet $A = \{1, 2, \ldots, 9\}$. We call a string S over A a *blue* string, if and only if the string consisting of the odd-positioned symbols in S is the reverse of the string consisting of the even-positioned symbols in S. For example $S = 14233241$ is a blue string since the substring of the odd-positioned symbols is 1234 which is the reverse of the substring of the even-positioned symbols, i.e., 4321.

Design a Turing machine which accepts all blue strings over A. You do not need to provide a formal description of the Turing machine but your description has to be detailed enough to explain every possible step of a computation.

Exercise 4: Bonus Question *(Points)*

Consider a push-down automata with two stacks instead of one. Can it be as powerful as a Turing machine? Give an intuitive explanation?
Hint: Try to show how can the Two-Stack PDA simulate the TM tape.