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Theoretical Computer Science - Bridging Course
Exercise Sheet 6

Due: Tuesday, 25th of November 2025, 12:00 pm

Exercise 1: Constructing TMs (continued) *(2+2 Points)*

Construct a Turing machine that decides on the languages:

(a) $C_1 = \{a^i b^j c^k | i - j = k \text{ and } i, j, k \geq 1\}$

(b) $C_2 = \{a^i b^j c^k | i \times j = k \text{ and } i, j, k \geq 1\}$

NB: A high level description is enough.

Exercise 2: A Better TM Variant *(3+2+2+1 Points)*

Let $\Sigma = \{0, 1\}$. For a string $s = s_1 s_2 \dots s_n$ with $s_i \in \Sigma$, let $s^R = s_n s_{n-1} \dots s_1$ be the *reversed* string. *Palindromes* are strings s for which $s = s^R$. Then $L = \{sas^R \mid s \in \Sigma^*, a \in \Sigma \cup \{\varepsilon\}\}$ is the language of all palindromes over Σ.

(a) Give a state diagram of a Turing machine recognizing L.

(b) Give the maximum number (or a close upper bound for the number) of head movements your Turing machine makes until it halts, if started with an input string $s \in \Sigma^*$ of length $|s| = n$ on its tape.

(c) Describe (informally) the behavior of a 2-tape Turing machine which recognizes L and uses significantly fewer head movements on long inputs than your 1-tape Turing machine.

(d) Give the maximum number (or a close upper bound for the number) of head movements your Turing machine makes on any of the two tapes until it halts, if started with an input string $s \in \Sigma^*$ of length $|s| = n$ on the first tape.

Exercise 3: Decidable Problems *(3+3+2 Points)*

(a) Show that the following languages are decidable.

- $A = \{\langle R, S\rangle \mid R \text{ and } S \text{ are regular expressions and } L(R) \subseteq L(S)\}$.
- $B = \{\langle G\rangle \mid G \text{ is a CFG over } \{0, 1\} \text{ and } 1^* \cap L(G) \neq \phi\}$.
  Hint:Use the fact that a language $C \cap R$ is context free for some context free language C and regular language R.

(b) Consider a decidable language L. Show that its complement $\overline{L}$ is also decidable.